# Interaction Between Parallel Underground Openings

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ABSTRACT: The paper involves a parametric study for investigating the interaction between parallel underground openings that have conventional shapes. The parameters of the study are related to problem geometry and in situ stress conditions. For this purpose, a two-dimensional finite element program with elasto-plastic stress analysis capability is used. The effects of size and relative position of openings on the interaction are investigated. The openings are located in a hydrostatic or an anisotropic in situ stress field. The results indicate that almost all the parameters studied affect the interaction of parallel openings in one way or another.

## 1 INTRODUCTION

Interaction between underground openings is one of the complex problems that a mining engineer encounters in the design of mine excavations. In many cases, the interaction among the mine openings is three dimensional and three-dimensional methods are required for studying such situations accurately. Yet, in some instances, when the longitudinal axes of openings are parallel to each other and also to one of the principal components of in situ stress field, the interaction problem becomes two dimensional and, then, a much simpler plane- strain analysis may be sufficient, to analyze it. In fact, a number of researchers have investigated the distribution of stresses around parallel underground openings by two-dimensional elastic - stress analyses. Unfortunately, available closed-form solutions involve only circular openings, and the opening shapes commonly employed in mining, therefore, can only be studied by numerical stress analysis.

In this study, some important aspects of the interaction phenomenon occurring between closely located and unsupported parallel underground openings are investigated. „Firstly, the approaches commonly employed for siting of parallel tunnels or underground galleries are summarized. Then, secondly, the mathematical or closed-form solutions available for the stresses around parallel tunnels are reviewed. Thirdly, important features of a parametric study for the analysis of interaction problem are described. Finally, the significant results of the study are presented and their implications are discussed.

## 2 SITING OF PARALLEL OPENINGS

Bieniawski (1984) considers the interaction between adjacent openings as one of the principal factors affecting stability of mines and tunnels. Therefore, in the design of parallel underground openings excavated close to each other, the effect their interaction on the overall stability should be taken into consideration. Siting of parallel underground galleries is sometimes a formidable task for mining engineers. For instance, deciding on the degree of allowable interaction between the openings and determining the size of a safe pillar between the neighboring openings are among the interesting problems of rock engineering.

Of course, there are always rules of thumb for practicing engineers. For example, according to US Army Corps of Engineers (1978), when two or more parallel tunnels are planned, the minimum width of a pillar to be left between unsupported tunnels should be 1 to 1.3 tunnel diameters in competent rock and 3 diameters or more in poor quality rock masses.

In addition, there are theoretically more reliable approaches. In this respect, the concept of "the zone of influence of an excavation" proposed by Bray (1987) has been very useful. When an underground opening is excavated, the in situ stress field is disturbed in such a way that the disturbance is largest in the close proximity of the opening and it diminishes rapidly away from the opening. The region surrounding the opening within which the disturbance is significant is called as "the zone of influence" (Bray 1987). The dimensions of the zone of influence depend on opening geometry (i.e. size and shape) and in situ stresses. This concept can be used for siting two or more excavations in close proximity. If none of the zones of influence overlap each other, the interaction between the excavations is virtually negligible. As a matter of fact, the excavations of which boundaries are located outside of one another's zones of influence can be designed by ignoring the presence of all others (Brady & Brown 1993). Using closed-form elastic solutions for stresses, Bray (1987) derived and presented practical expressions for determining the width and height of the zone of influence for circular and elliptical openings. According to these expressions, in a hydrostatic stress field, the mechanical interaction between two circular openings of the same size can be considered insignificant if the width of pillar left between them is greater than two tunnel diameters. Similar findings were reported by Ohaboussi & Ranken (1977), who studied the interaction between two parallel circular tunnels by finite element analyses.

## 3 STRESSES AROUND PARALLEL UNDERGROUND OPENINGS

Jeffrey (1920) was the first researcher who developed plane elasticity equations in bipolar coordinates and a stress function applicable to the problem involving an infinite plate with two circular holes. Although he did not present a solution for this case, it was the first theoretical approach applicable to the problem of parallel tunnels, and a number of others followed it.

Howland (1935) investigated the stress distribution around an infinite row of equal size circular holes spaced equally in an infinite elastic plate. The plate was subjected to a uniaxial stress field (either parallel or perpendicular to the line of holes). By using principle of superposition, solution for biaxial stress fields can be obtained for this problem. Howland & Knight (1939) presented stress functions for the problems involving equal size circular holes (e.g. one or two pairs, a single row or double rows, etc.). However, no explicit solution was given for the particular cases considered.

In 1948, Ling developed a widely referenced solution (in bipolar coordinates) for the stresses in a plate containing two equal circular holes with a variable distance between them. He considered three stress fields: uniaxial stress parallel and perpendicular to the line of centers and equal stresses in all directions. About two decades later, Haddon (1967), using the conformai mapping and complex variable techniques, presented a solution for stresses around two unequal circular holes in an infinite plate. The plate was subjected to a uniaxial stress field with a variable inclination to the line of centers. This interesting solution, though tractable, involves too many terms and coefficients.

More recently, Gerçek (1988, 1996) presented a solution for boundary stresses for two parallel circular tunnels located in a biaxial in situ stress field. It was obtained by superposing the solutions developed by Ling (1948). In addition, Zimmerman (1988, 1991) gave approximate expressions for stress concentrations in an infinite elastic plate containing two circular holes.

Although the solutions mentioned above are not exhaustive, all the available closed-form solutions involve only elastic behavior and circular openings; for that reason, the openings with conventional shapes and excavated in elasto-plastic rock masses can only be studied by numerical stress analysis. In the following sections, a summary of such a study is presented.

## 4 STRESS AND STABILITY ANALYSES

The important aspects of the stress and stability analyses are summarized below.

### 4.1 Numerical Stress Analysis Program

In the study, Phase$^2$ computer program is employed for the stability analysis of parallel underground openings. Phase$^2$ (v5.0) is a two-dimensional elasto-plastic finite element program for calculating stresses and displacements around underground or surface excavations (Rocscience 2004). It can be used for studying and geomechanical evaluation of a wide range of mining and civil engineering projects (e.g. complex underground mining excavations or tunneling problems, surface excavations such as open pit mines, and slopes in rock or soil). The program is well suited for parametric studies because of its friendly capability for creating models (including automatic mesh generation and refinement), efficiency in performing nonlinear stress analysis, and versatility in interpreting results with interface tools. Further information about Phase$^2$ can be found elsewhere (Rocscience 2004).

### 4.2 Problem Geometry

The problem geometry is depicted in Figure 1. It involves two parallel underground openings. They are of conventional shape having an ached roof and a flat floor. The width-to-height ratios of the openings are Wi/Hi and W2/H2. The minimum width of the pillar left between the openings is $W_P$. The angle a characterizes the orientation of the openings with respect to each other. In the study the following parameters are considered: W1/H1 = $W_2/H_2$ = 2; $W_1/W_2$ = 1 and 2; *WrWi* =0.5; a = 0, n/4, and n/2.

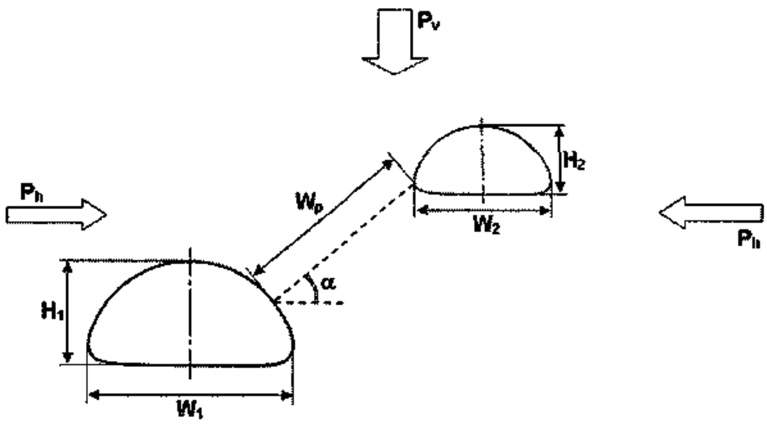



Figure 1. Geometrical parameters and in situ stress field considered in the numerical analyses.

### 4.3 Rock Mass Properties

In the analyses, the rock mass behavior is assumed to be elasto-plastic. For the characterization of rock mass strength, the following version of the Hoek-Brown empirical failure criterion (Hoek & Brown 1997) is considered:

$$\sigma_1' = \sigma_3' + \sigma_{ci} (m_b \sigma_3' / \sigma_{ci} + s)^{0.5} \qquad [1]$$

where oV and $o_3$' are the maximum and minimum principal effective stresses, respectively, at failure, and Od is the uniaxial compressive strength of intact rock. The Hoek-Brown constants $m_b$ and s depend on the quality of the rock mass, and they can be estimated by the following empirical expressions involving the Geological Strength Index (Hoek & Brown 1997) or GSI:

$$m_b = m_i \exp[(GSI-100)/28)] \qquad [2.a]$$

$$s = \exp[(GSI-100)/9)] \qquad [2.b]$$

where mi is the material constant for intact rock. In the numerical analyses, the following strength parameters have been used for the intact rock material: mi = 10 and a^ = 75 MPa. Also, the following Hoek-Brown strength parameters have been used for the original rock mass: GSI = 70, $m_b$ = 3.43, and s = 0.036.

For the failed rock mass, the following expressions suggested by Ribacchi (2000) are used.

$$m_r = 0.65\ mi, \qquad [3.a]$$

$$s_r = 0.04\ s \qquad [3.b]$$

Then, the strength parameters are $m_r$ = 2.23 and $s_r$ = 0.00143 and the dilatation parameter is assumed to be zero.

The deformation modulus of the rock mass is estimated by the following empirical expression suggested by Hoek & Brown (1997):

$$E_m\,(GPa) = [\sigma_{ci}\,(MPa) / 100]^{0.5}\ 10^{(GSI-10)/40} \qquad [4]$$

Thus, the elastic properties for the rock mass are as follows: $E_m$ = 27.4 GPa, v (Poisson's ratio) = 0.25.

### 4.4 In **Situ** Stress **Field**

The results of numerous in situ stress measurements carried out at various regions of the world indicate that, unless there is any significant geological effect, the principal in situ stresses do not deviate very much from the vertical and horizontal directions (Amadei & Stephansson 1997). In the study, too, it is assumed that the principal components of the in situ stress field act in vertical, horizontal, and axial (longitudinal) directions with respect to the parallel openings, and they are $P_v$, $P_b$, and $P_z$, respectively (Fig. 1).

In addition, both isotropic (i e hydrostatic) and anisotropic in situ stress fields have been considered in the numerical analyses

4 4 1 *W opic in situ stresses*

Consideration of hydrostatic m situ stress field m the stress and stability analyses of underground openings facilitates solutions and interpretation of results For a circular opening, if failure occurs, the thickness of failure zone is expected to be constant m a hydrostatic stress field In addition, it has long been recognized that if failure occurs around a non-circular underground opening located m a hydrostatic in situ stress field, the thickness of failure zone is larger at the parts of boundary where the radius of curvature is larger or vice versa

In the study, the hydrostatic in situ stress is taken as $P_v = P_h = P_z = P_0 = 20$ MPa to cause a substantial zone of failure around the openings According to the closed form solution of Brown et al (1984) for a circular tunnel excavated m a rock mass with given strength parameters and $P_0 = 20$ MPa, the thickness of the failure zone is about 14 5 % of the tunnel diameter

4 4 2 *Anisotropic in situ stresses*

The in situ stress measurements reported in the literature also indicate that the magnitudes and order of principal components are quite variable (Amadei & Stephansson 1997) In fact, anisotropic in situ stress field is a common occurrence throughout the world

In this study, in order to impose a considerable anisotropy for the in situ stress field, its major, intermediate, and minor principal components are chosen as Pi = 30 MPa, $P_2$ = 20 MPa, and $P_3$ = 10 MPa, respectively The anisotropic m situ stress combinations considered in the analyses are given in Table 1

Table 1 The anisotropic in situ stress combinations considered in the study

| Case | P, (MPa) | Ph(MPa) | **P,(MPa)** | **k = P,/P,,** |
|---|---|---|---|---|
| Al | 10 | 20 | **30** | **20** |
| A2 | 10 | 30 | **20** | **30** |
| A3 | 20 | **10** | **30** | **05** |
| A4 | 20 | 30 | **10** | **15** |

These in situ stress states represent equivalent stress points with respect to the the Hoek-Brown yield surface In other words, in the principal in situ stress space, the stress points are on the same octahedral plane (Eqn 5a), they are located at equal distances from the hydrostatic stress axis (Eqn 5b), and they represent pure shear (Eqn 5c) Then the following expressions can be given for these stress points

$$I_1 = P_v + P_h + P_z = 60 \text{ MPa} \quad [5a]$$

$$J_2^{1/2} = \{[(P_v-P_h)^2+(P_h-P_z)^2+(P_z-P_v)^2]/6\}^{1/2} = 10 \text{ MPa} \quad [5b]$$

$$\theta = \{\text{arcsm} [-1\ 5\ J_3 (3 / J_2^3)^{1/2}]\} / 3 = 0 \quad [5c]$$

where $I_1$ = first invariant of the m situ stress tensor, J2 and J3 = second and third invariants, respectively, of the deviatonc m situ stress tensor, and 0 = Lode angle

4 5 Evaluation of Stability

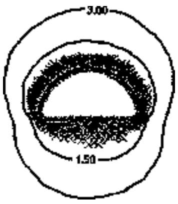
The results of a stress analysis may be evaluated in many ways, e g by looking at the distribution of pnncipal components of induced stresses or displacements occurring around the openings One of the more meaningful approaches is to evaluate the distribution of local safety factors around the underground openings along with the failure zones if they develop This is an indication of how the excavations change the stability picture withm the immediate neighborhood of the openings Also, in comparison of situations with different m situ stress fields, one should be very careful since the results may be misleading if the initial conditions (1 e m situ stress states) are not equivalent

5 RESULTS OF THE ANALYSES

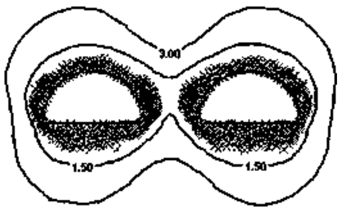
In this section, significant results of the numerical analyses are presented, firstly, for the hydrostatic in situ stress field and, then, for the anisotropic in situ stress cases

5 1 Isotropic In Situ Stresses

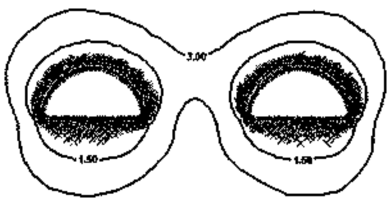
Before studying the interaction problem, the stability picture of a single isolated opening has been determined for hydrostatic in situ stresses For this purpose, the distribution of safety factors around the opening and the size of failure zone are examined (Fig 2a) Then, the effect of pillar width ($W_p$) on the interaction between two parallel openings of the same size (1 e Wi = $W_2$) is investigated for the same in situ stresses The cases with $W_p$/Wi =0 5, 1, and 15 are considered (Figs 2b, 2c, and 2 d, respectively) As expected, for the opening shapes considered, practically no interaction exists between the parallel openings when $W_p$/W| > 1 5 (compare Figs 2a and 2d) Also, it is noted that the interaction is minimal when Wp/Wi = 1 In order to study the interaction more closely, the pillar width is taken as one-half of the opening width (1 e $W_p$/Wi = 0 5) for the remainder of the study



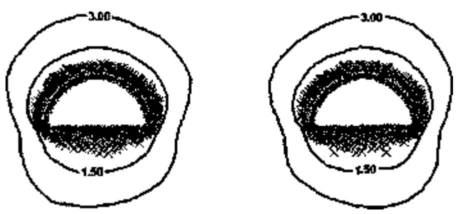

a. Single isolated opening.




b. Parallel openings with $W_P$ / W, = 0.5.




c. Parallel openings with $W_P$/ W, = 1.

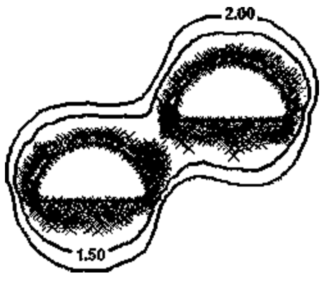



d. Parallel openings with $W_P$ / Wj = 1.5.

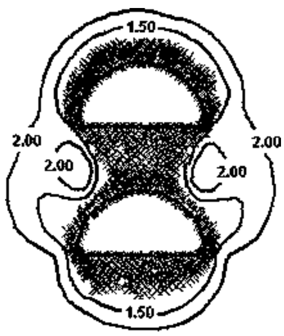
Figure 2. Effect of the pillar width on the degree of interaction.

Then, the effect of the position of openings with respect to each other is examined. Three different alignments of openings are considered: horizontal alignment (a = 0), diagonal alignment (a = n/4), and vertical alignment (a = *nil)*. The distribution of safety factors and the failure zones are illustrated in Figure 3. It is noted that, for such a close siting, the vertical alignment of openings is the worst position in terms of stability; actually, this is to be expected since the parts of opening boundaries with larger radius of curvature are opposite to one another.Also, for the hydrostatic in situ stress field, the interaction between parallel openings with different sizes is studied. The dimensions of one of the openings is taken as one-half of those of the other (i.e. Wi/$W_2$ = 2). In this respect, the same (i.e. horizontal diagonal, and vertical) alignments apply with one dih.rence. There are two versions of the vertical alignment: the smaller opening is situated just above (a = Jt/2) or just below (a = -it/2) the larger one. In all cases, the width of pillar between the openings is taken as equal to the width of smaller opening ($W_P$ = W2). The results of the analyses are depicted in Figures 4a to4d.


a. Horizontal alignment (a = 0).

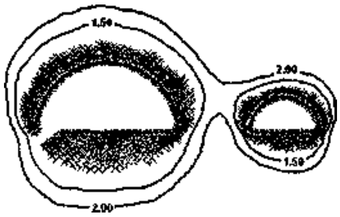



b. Diagonal alignment (a = 7t/4).

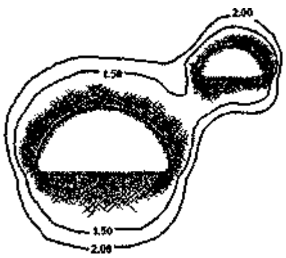



c. Vertical alignment (a = *itfl)*.

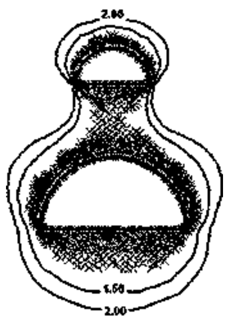
Figure 3. The effect of positions of openings on the interaction in a hydrostatic in situ stress field.



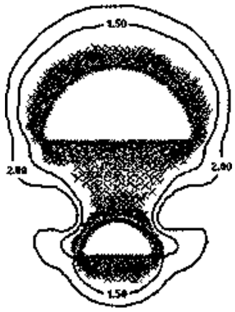
a. Horizontal alignment (a = 0).



b. Diagonal alignment (a = *n* /4).



c. Vertical alignment (ot = *n 12).*



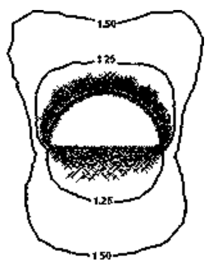
d. Vertical alignment (a = - *n 11).*

Figure 4. Interaction between openings with different sizes in a hydrostatic in situ stress field.

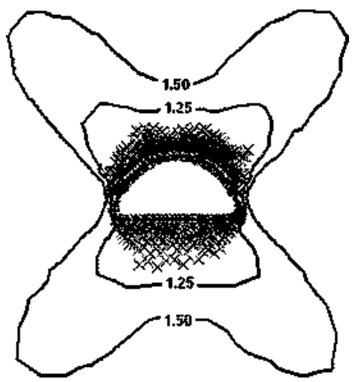
Similar to the situation with openings of equal size, in the cases involving excavations with different sizes, the vertical alignment positions (Figs. 4c and 4d) result in worse stability conditions than those of the other cases. Again, in these cases, the parts of opening boundaries with larger radius of curvature are opposite to one another and, therefore, almost the entire section of the pillar left between the openings is subjected to high stresses.

5.2 Anisotropic In Situ Stresses

Similarly, stability of a single isolated opening has been determined for the anisotropic in situ stress cases given in Table 1. Contours of safety factor and failed regions around the single openings are determined. The results of Cases Al and A2 (i.e. $P_v$ is the minimum component of the in situ stresses) are shown in Figure 6 while the results of Cases A3 and A4 (i.e. Pv is the intermediate component of the in situ stresses) are shown in Figure 7.

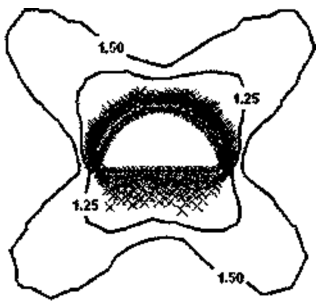


a. Case Al ($P_v$ = 10 MPa, $P_h$ = 20 MPa, $P_z$ = 30 MPa).




b. Case A2 ($P_v$ = 10 MPa, $P_h$ = 30 MPa, $P_2$ = 20 MPa).

Figure 5. Contours of safety factor and failed regions around the single openings when $P_v$ is the minimum component.



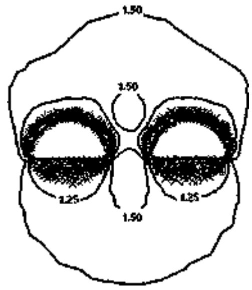

a. Case A3 (P, = 20 MPa, $P_h$ = 10 MPa, $P_2$ = 30 MPa).


b. Case A4 ($P_v$ = 20 MPa, $P_h$ = 30 MPa, $P_2$ = 10 MPa).

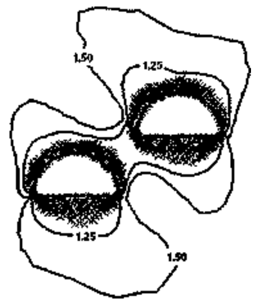
Figure 6. Contours of safety factor and failed regions around the single openings when $P_v$ is the intermediate component.

According to these initial analyses, when $P_v$ is the minimum in situ stress component, orienting the opening parallel to the larger of the horizontal stresses (i.e. Case A1) results in considerable smaller yield or overstressed region, compared to me reverse situation (Fig. 5). This is in agreement with the recommendations and reported cases encountered in the literature. However, when $P_v$ is the intermediate in situ stress component, somewhat opposite results have been obtained. In this case, orienting the opening parallel to the smaller of the horizontal stresses (i.e. Case A4) has resulted in significant decrease in the size of overstressed region (Fig. 6). Similar findings were also reported by Gerçek & Genis (1999).

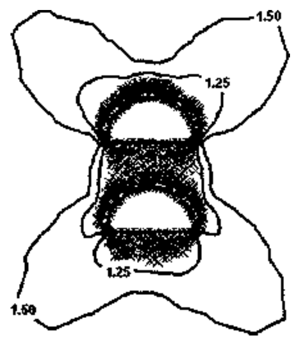
The implication of these phenomena for the interaction problem is that any in situ stress condition or opening orientation which is unfavorable for a single opening will also be undesirable for closely spaced parallel openings. Indeed, the outcome of analyses involving anisotropic in situ stresses has confirmed the findings. For that reason, when $P_v$ is the minimum component, only Case Al is shown in Figure 7 since Case A2 is already known to be worse in terms of stability. Among the three different positions, again, the vertical alignment of openings is the most unfavorable one (Figs. 7a-c).




a. Horizontal alignment (a = 0).



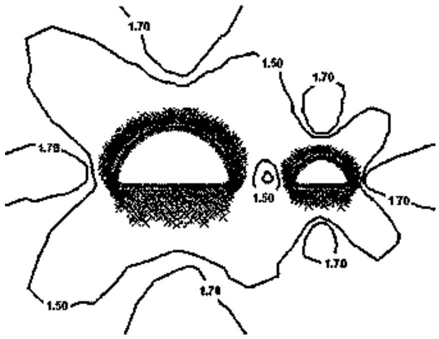

b. Diagonal alignment (a = 3t/4).



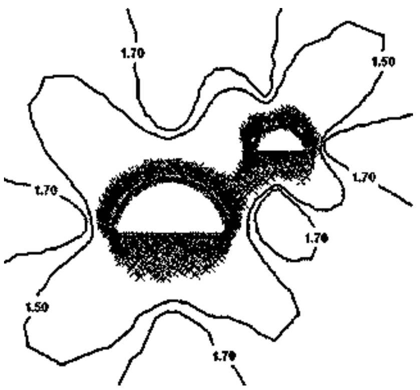

c. Vertical alignment (a = n/2).

Figure 7. Interaction between openings in an anisotropic in situ stress field (Case Al; $P_v$ = 10 MPa, $P_h$ = 20 MPa, $P_z$ = 30 MPa).

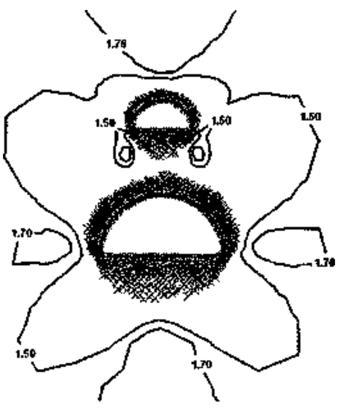
Finally, the other anisotropic in situ stress conditions, in which $P_v$ is the intermediate component, (i.e. Cases A3 and A4) are considered. In terms of stability, Case A3 is worse than Case A4. In Figure 8, results of the analyses for parallel openings with different sizes are shown for Case A3. It is interesting to note that, as regards to stability, the diagonal alignment of openings is the least desirable one (Figs. 8a-c).



a. Horizontal alignment (a = 0).



b. Diagonal alignment (a = 7t/4).



c. Vertical alignment (a = *TU2).*

Figure 8. Interaction between openings in an anisotropic in situ stress field (Case A3; P, = 20 MPa, $P_h$ = 10MPa, $P_z$ = 30MPa).

## 6 CONCLUSION

Interaction between closely spaced and parallel underground openings with complicated geometry can be studied by only numerical stress analysis. In addition to the in situ stresses field, the geometry (size and shape) of openings and their position with respect to each other are important factors that affect the degree of interaction.

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