

## Thin Spray-on Liners for Underground Rock Support

D.D.Tannant

*Department of Civil & Environmental Engineering, University of Alberta, Edmonton, Canada*

**ABSTRACT:** Rapid setting, thin, spray-on polymeric liner materials for underground rock support are being tested in Canada. Thin polymer liners have performance characteristics that lie between those of shotcrete and mesh. They are a welcome addition to the 'tool box' of support types and have a role to play where rapid application rates and areal support of rock are needed. A continuous liner that is firmly adhered to the rock creates effective rock support. Various approaches are used to examine the load capacity of a liner. Interpretation of the available test data and introduction of simple support models show that the two likely failure modes are adhesion loss and tensile or shear rupture of the membrane. Different failure modes occur depending on the relative tensile and adhesive strengths of the liner and the anticipated magnitude of the rock displacements.

### 1 INTRODUCTION

Thin spray-on liners are a new form of rock support that is receiving increasing attention by various mines around the world. Various liner materials are currently being developed and tested in Canadian mines. They can all be generically classified as multi-component polymeric materials. Thin polymer liners have applications in hard-rock mines as a replacement for either wire mesh or shotcrete. They function well as the areal support component in a support system that also incorporates rock bolts.

This paper gives a historical overview of the development and testing of spray-on liner materials and discusses various mechanisms by which thin liners function to support and stabilize underground excavations in rock. Important liner properties include the tensile strength and the adhesion to the rock, which control the liner's load capacity, and the ultimate strain, which controls the liner's displacement capacity.

Various factors to consider when using thin spray-on liners, including advantages and disadvantages when compared to welded-wire mesh or shotcrete are reviewed. Simple models that illustrate various support functions and the important design parameters for thin liners operating under various conditions are presented. Finally, a review is presented of current thin liner use in Canadian mines, outlining successes and future challenges.

### 2 DEVELOPMENT OF THIN LINER SUPPORT

The installation of conventional rock bolts and wire mesh is labour intensive and time consuming. In addition, underground personnel are frequently injured while installing rock support. While the installation of rock bolts or other tendon support elements can be mechanized, mesh installation still requires manual labour. One method for overcoming some shortcomings of mesh is the use of shotcrete, in particular, steel fibre reinforced shotcrete. While the use of shotcrete rapidly gained acceptance within many Canadian mines in the 1990's there are still problems associated with the logistics of transporting large quantities of shotcrete *materials to* active headings far underground. In addition, it was noticed that deeper drifts in many Canadian mines underwent substantial deformations. These deformations exceeded the displacement capacity of the shotcrete rendering the shotcrete itself a hazard.

As an alternative to rock bolts and mesh or shotcrete, MIROC (Mining Industry Research Organization of Canada) began an investigation of rapid setting, thin, spray-on liner materials for ground support. The first tests on thin spray-on liner rock support technology were initiated in Canada in the late 1980's (Archibald et al. 1992). Initial research led to the development of a polyurethane based product call Mineguard™. Modifications to the chemical formulation of Mineguard and extensive laboratory testing continued throughout the 1990's (Archibald et al. 1997, Archibald & Lausch 1999).

In the 1990's, Canada's largest nickel mining company, INCO Ltd., embarked on a strategy to move toward robotic mining methods. The use of a thin spray-on liner for underground rock support offered INCO numerous advantages in terms of speed of application and minimizing transportation of materials. INCO thus became a key advocate of thin liner research and testing and sponsored numerous laboratory and field tests using Mineguard (Figure 1) throughout the 1990's (Espley et al. 1995, 1996, Espley-Boudreau 1999, Tannant 1997, Tannant et al. 1999). In the late 1990's, these tests also included a new product based on hybrid polyurethane/polyurea mixture called Rockguard.



Figure 1 Manual application of a thin polyurethane (Mineguard) liner to rock.

Meanwhile in 1996, researchers from South Africa began exploring the use of another thin liner product that was latex-based. This product was known as Everbond (Wojno & Kuijpers 1997) and has since evolved into another product called Evermine. Researchers in Australia have also been exploring the use of thin liners for rock support and have conducted field tests in Western Australia.

By the mid to late 1990's news of spray-on liners being used in Canada's hard-rock underground mines reached many other interested manufacturers and vendors of a wide variety of spray-on products. Falconbridge Ltd. (another large Canadian mining company) began its own research effort to find appropriate liner materials. While many products were tested, it was found that most did not possess adequate physical or chemical properties. One product called TekFlex (a water-based, polymer modified cementitious material) was found to show promise and field trials of this material were initiated in cut and fill stopes (Pritchard et al. 1999, 2001).

A variety of new products are in the development and testing stages. These include a polyurea-based product called RockWeb and a methacrylate-based product called Masterseal 840R01 or Superskin (Spearing & Champa 2000). As of 2000, there are about six different manufacturers of spray-on materials for thin liners that are competing in the market for underground rock support in Canada.

There are currently about 55 mines around the world that are considering the use of thin liners for rock support. The greatest interest is in North America, Australia, and South Africa. Given that thin liner technology is still in its infancy, it is likely that other products will come forward for testing and evaluation. Good liner materials adhere tenaciously to the rock surface, cure quickly, and have high tensile strength. This paper does not focus on comparing one product versus another. Refer to the publications listed in the references for properties of specific liner materials. Instead, the remainder of the paper examines design issues related to thin liner support technologies.

### 3 ROCK SUPPORT PROVIDED BY THIN LINERS

A principal objective of support is to assist the rock mass in supporting itself. It is difficult for a support system to hold up the dead weight of rock once the rock mass has loosened (Hoek & Brown 1980). This is particularly true when using thin liners, because they have a limited load capacity. In jointed or fractured rock masses, a thin liner prevents the rock mass from dilating, loosening and unraveling, thus forcing fragments of the rock mass to interact with each other creating a stable beam or arch of rock. To be effective at helping establish a stable zone of rock, a liner must be able to limit the kinematic movement of individual rock blocks. If conditions allow the rock mass to loosen excessively, then the liner's function can switch to retaining the loose rock in place between rock bolts.

Conventional support in the hard-rock mining industry makes use of rock bolts or other tendon support to hold large key-blocks in place while wire mesh is used to retain the small rock pieces between the tendons. In some cases, shotcrete is used in a dual role for supporting both larger key-blocks as well as smaller pieces of loose rock.

Most support design focuses on the load capacity of the support. It is equally important to consider the support's displacement capacity, especially in situations where large ground convergence and significant relative displacements or shear displacements between adjacent rock blocks are expected. Only through knowledge of the displacement capacity of various support types can proper design and selection of support be made for a given application.

### 3.1 Displacement capacities of areal support

Shotcrete, polymer liners, and steel mesh mobilize support resistance at different displacements. Materials that are sprayed onto the rock such as shotcrete or liners are able to generate support resistance at small rock deformations (in the order of millimetres). Mesh is a truly passive support and requires substantial displacement (in the order of 100's of millimeters) before it offers a support resistance (Tannant 1995, Tannant et al. 1997). Mesh is effective at catching and hold small falls of rock, but it provides minimal resistance to the initiation of the rockfall itself. Sprayed materials operate differently because they are able to offer support resistance at small displacements. Therefore, they can prevent rockfalls from happening in the first place.

Shotcrete, especially reinforced shotcrete, can generate much higher support resistance than thin polymer liners. However, in situations where large ground convergence occurs, the more flexible thin liners may provide superior support over the full range of rock deformations. For example, insitu pull tests using a 250mm diameter plate pulled through 70 to 100mm thick steel fibre-reinforced shotcrete showed that the shotcrete could only sustain 5 to 10mm of relative displacement before the shotcrete ruptured and failed (O'Donnell & Tannant 1998).

Laboratory pull tests on 1.5m square panels made from concrete blocks coated with 50 to 60mm of steel-fibre reinforced shotcrete also showed a limited displacement capacity (Tannant & Kaiser 1997, Kaiser & Tannant 1997). The shotcrete panels attained peak strength and fractured after relative displacements of 5 to 10mm. In comparison, concrete blocks coated by a polyurethane membrane tested in similar conditions did not reach peak load until 40 to 50mm of displacement and the load was maintained for up to 100mm of displacement (Tannant 1997).

Figure 2 shows schematically the different load-displacement performance for various areal support types. Liners are expected to have performance characteristics that lie between mesh and shotcrete.

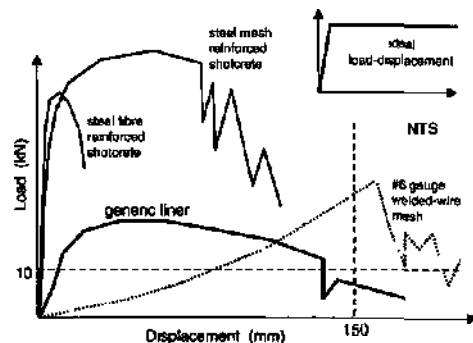


Figure 2 Load versus displacement capacities from pull tests on various areal support types.

### 3.2 Support of small loose rocks - Glue action

A thin liner may simply 'glue' or bond loose pieces of rock to adjacent competent rock. No other conventional support type is designed to act like glue between adjacent rocks. One exception may be rock masses that are grouted before excavation occurs.

The mass of loose rock that can be safely held in place depends on the liner adhesion to the rock and the polymer's tensile or shear strength. Another parameter, which is difficult to determine, is the adhesive bond width. The effective bond width dictates the area over which the membrane acts while carrying a tensile load. The estimated properties for a polyurethane-based liner and a liner made from polymer-modified cement are listed in Table 1. Note these values are rough approximations at best.

Table 1 Typical liner material properties assuming a 4mm thickness

Polyurethane	
Tensile strength, $\sigma_t$	8MPa
Adhesive strength, $\sigma_a$	1MPa
Bond width, $w_b$	5mm
Polymer cement (after 4 hrs)	
Tensile strength, $\sigma_t$	1MPa
Adhesive strength, $\sigma_a$	1MPa
Bond width, $w_b$	8mm

Two support functions and their related liner failure modes are shown in Figure 3. Both cases rely on penetration of polymer material into gaps between loose rock blocks. Although penetration into real fractures or joints can occur in the field, it should be negligible. If the liner design relies on shear rupture through polymer material infilling open fractures or joints (Figure 3a) or adhesion between suspended loose rock and competent rock (Figure 3b), then it is likely that poor site preparation practices (scaling) have been used. Field evidence suggests that careful site preparation is critical to the success of a thin liner and loose rock should be scaled down before application of the liner. Nevertheless, the simple models shown in Figure 3 can be used to evaluate, in a general manner, the holding capacity of a spray-on material that acts like glue.

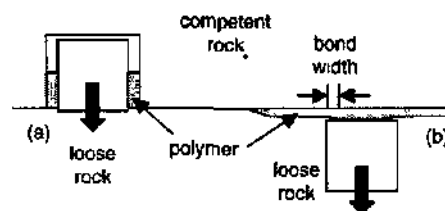


Figure 3 Possible support functions involving small loose rocks with an assumed shape of a 100mm cube.

In situations where open joints or fractures exist it is possible for the sprayed polymer to penetrate into the loose rock. Figure 3a show a simple model of a loose cube-shaped rock bonded to competent rock by a polymer around its four sides. The force required to remove the block from the competent rock depends on the depth that polymer penetrates the gap and the shear strength of the polymer. Based on laboratory conditions where sprayed polyurethane was used to coat concrete blocks separated by gaps of 1 to 2mm, the depth of consistent penetration is limited to less than about 20 to 50mm. A penetration,  $d_p$  of 20mm will be assumed here. Material testing for polymer membranes is typically performed to measure tensile properties; therefore the shear strengths of these materials are largely unknown. The shear strength of a polyurethane membrane is likely to be similar to its tensile strength, assumed to be about 8MPa. The force,  $F$  needed to pull the loose bonded rock (with perimeter length  $L$ ) away from the competent rock is:

$$F = L \cdot d_p \cdot \sigma_t = 0.4\text{m} \cdot 0.020\text{m} \cdot 8\text{MPa} = 64\text{kN} \quad (1)$$

Even if the polymer penetrated on only one side of the loose block the force required to remove the rock (16kN) would be much larger the weight of the rock. For example, if the rock were a 100mm cube, it would weigh only about 26N. Hence, the support capacity is nearly three orders of magnitude higher than the block's weight.

By assuming another simplistic model of a loose rock glued to competent rock over a 100mm by 100mm bonded area, one can determine the effectiveness of a polymer material that has penetrated a gap between a loose rock and competent rock (Figure 3b). In this scenario, the polymer adhesion is the weak link. Although the polymer is assumed to fully cover the contact between the rocks, the adhesive strength is not mobilized over the whole area because progressive failure of the adhesive bond would occur. If it is assumed that the whole perimeter of the rock, over an effective bond width of 5mm, carries load before adhesive failure initiates, then the force needed to dislodge the rock,  $F$  is given by:

$$F = L \cdot w_b \cdot \sigma_a = 0.4\text{m} \cdot 0.005\text{m} \cdot 1\text{MPa} = 2\text{kN} \quad (2)$$

This is more than sufficient force to hold in place the weight of a small rock. Even if eccentric loading acts on the rock such that only one side of the rock is loaded in tension, the force holding the rock in place would be 0.5kN, which is much greater than the weight of the small rock.

The two simple models show that a thin polymer is quite effective at holding small rocks in place if sufficient polymer material is able to fill the gap between the loose and competent rock. This ability is clearly evident in laboratory tests when loose

rocks are bonded together by any of the polymer materials. Once bonded together the individual rocks are virtually impossible to tear apart by hand.

However, for actual liner design purposes in a drift with careful scaling it may be best to ignore any possible penetration of the polymer into fractures or joints. Any penetration that does occur will likely improve the support capacity of the liner. But it is important to remember that the overall objective is to have tight, sound rock present before the liner is sprayed. Failure to do so means that a sufficient portion of the rock mass's self-support capability has already been compromised before the liner is applied.

### 3.3 Support for drifts - Membrane action

At scales larger than that depicted in Figure 3, i.e., for general rock support across the back of a drift, the liner performs a support role by resisting relative movement between individual blocks of rock and possibly acting as a suspended membrane in tension carrying rock loads (Espley et al. 1999). A thin liner applied to the excavation surface, especially at the locations of fractures and discontinuities, is effective at resisting relative movement between individual blocks of rock. The liner performs this function through a combination of a gluing action as described earlier and a membrane action. The membrane action becomes more important if the liner is forced to experience larger deformations.

A liner is most effective when applied to the rock before significant movement takes place. In some cases, the liner can 'lock' the blocks together keeping relative block displacements small (<1mm) and thus function to stabilize the rock mass around the excavation. In other cases, when rock mass conditions, stress levels, and the excavation geometry combine to generate larger rock deformations or convergence, a thin liner may not be able to suppress relative displacements and a zone of unstable rock will develop. Under these conditions, the liner typically acts like a deformable membrane to retain and hold the rock in place.

The support function that a thin liner may play depends on the amount of the rock that is involved and the magnitude of relative displacements between adjacent rock blocks. It is important to recognize that large convergence may not be a problem so long as the rock moves inward in a uniform manner.

### 3.4 Potential liner support failure modes

The model shown in Figure 4 can be used to analyze the support capacity of a liner with thickness  $t$  holding a loose rock block that undergoes either small or large displacements. The surface area of the block coated by the liner is assumed square in shape with width  $s$ . The block is assumed to move vertically

downward a distance  $d$  thus inducing stress in the liner. The first check is to determine whether the liner ruptures at small displacements due to either shear or diagonal tensile stresses around the perimeter of the block (Figure 5).

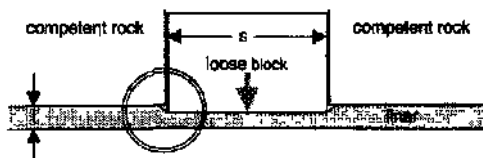


Figure 4 Model for rock support by a liner assuming a square block moving vertically downward.

### 3.5 Small deformations (<1mm relative rock movement)

Thin liners can resist shear displacements of up to a few millimetres. They achieve this by using a combination of shear, adhesive, and tensile strength. Relatively high liner stiffness is probably beneficial in this case. At small rock displacements the liner functions to prevent unraveling of small rock fragments, lock small rock blocks or wedges in place (key blocks), prevent loosening of the rock mass, mobilize interactions between rock blocks, and establish a stable arch of self-supporting rock. At small block displacements a thin liner acts like shotcrete in an active manner.



Figure 5 Liner failure modes at small block displacements caused by either shear rupture or diagonal tensile rupture.

The liner can fail in two modes Figure 5. It is assumed that failure of the adhesive bond does not occur. Given that a typical liner is only a few millimetres thick, direct shear failure or diagonal rupture of the membrane must occur within the first few millimetres of relative rock displacement. These two failure modes are most likely when the liner adhesive strength is similar to the tensile strength. Note that this is the situation for unreinforced shotcrete, which is why shear or diagonal tensile failure modes occur in shotcrete.

For the failure modes shown in Figure 5 the support capacity (expressed here as force per unit length around the block perimeter) is a simple function of the liner thickness and either the shear or tensile strength of the liner. As before, given lack of test data, the shear strength will be assumed equal to the tensile strength.

$$F = t \cdot \sigma, \text{ per metre} \quad (3)$$

Assuming a liner thickness of 4mm and tensile strengths of either 1 or 8MPa (Table 1), Equation 3 yields a support capacity in range of 4 to 32kN/m. If the block size was 1m by 1m, with a density of 2600kg/m<sup>3</sup>, then the liner could theoretically hold the weight of a block that was 0.6 to 5m high. Note, it is overly optimistic to expect a 4mm thick liner to hold up the weight of 5m of rock. In a real excavation the loading conditions would be irregular and would greatly reduce the membrane's support capacity.

From the geometry shown in Figure 5b, one could argue that for diagonal tensile rupture, the true thickness of the liner carrying stress is greater than the liner thickness by roughly  $1/\sin 45^\circ$ . However, given the uncertainty in the parameters, this effect is ignored.

While the approach presented here is illustrative, rigorous liner design is nearly impossible given the complicated geometry of the interacting rock blocks and the unknown nature of all the forces acting through the arch to stabilize rock mass. Clearly, sprayed polymer materials are capable of holding in place small rocks as demonstrated earlier. But support design for a whole drift should be a philosophy or approach that dictates the need to maintain the inherent rock mass strength. The application of a liner is just one of many activities that can be used in this regard. For example, careful blasting practices are important too. In blocky rock masses, the use of a polymer liner may aid in the development of a stable Voussoir beam.

Observations gathered from field and laboratory pull tests indicate that neither of the two failure modes depicted in Figure 5 are common for polyurethane liners.

### 3.6 Large deformations (=1mm relative rock movement)

When conducting large pull tests on liners, the block displacements observed at the peak load are typically much greater than the thickness of the membrane. This demonstrates that the membrane is able to deform and stretch before it fails. In order for significant stretching to occur, some adhesion loss must also occur, providing a debonded length of membrane for stretching. Therefore, adhesion loss followed by tensile rupture is an important process from a design point of view.

These observations are consistent with the physical properties and liner thicknesses in use today (Table 1). Using the data for a polyurethane liner, the force needed to shear through a liner is about the same as the force needed to rupture the liner in tension (Equation 3) and is 32kN/m for a 4mm thick liner. The force required to initiate adhesive

debonding ( $G_c$ ,  $\sigma_{ad}$ ) is 5kN/m. Therefore, when the adhesive strength to the rock is significantly less than the liner tensile strength and the effective bond width is roughly the same as the liner thickness, adhesive failure around the displaced rock must occur first.

When the adhesive strength is less than the tensile strength the liner adhesive bond may progressively fail around the displacing block. By debonding, a section of liner rotates and begins to act in tension to resist the weight of the moving block as shown in Figure 6. Under these conditions, the liner can tolerate relatively large block displacements. Force equilibrium can be achieved when the vertical component of the tensile forces acting in the liner equals the weight of the block (assuming no frictional resistance along the sides of the block).

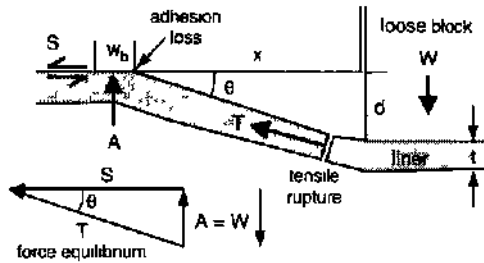


Figure 6 Interaction between liner adhesion and tensile strength to support the weight of a displaced block (only half of the model is shown).

The model first looks at the adhesive capacity of the membrane. If the block movement causes progressive adhesive failure, the debonding will progress away from the edge of the block (Figure 6). In doing so, the area over which the adhesion acts grows because the perimeter length increases. It is assumed that the area eventually becomes large enough to create an adhesive force  $A$  that satisfies force equilibrium with the weight of the block. The width of the debonded zone  $x$  at equilibrium or when tensile rupture occurs is calculated from:

$$A = 4\sigma_a (s + 2x)w_b = W \quad (4)$$

where  $W$  is the weight of the block,  $a$  is the average adhesive strength of the membrane acting over the effective bond width  $w_b$ , and  $s$  is the width of the block. Equation 4 can be used to determine the width of the debonded area.

Adhesive support from the liner has now been fully mobilized so attention can now turn to the tensile strength of the liner. It is reasonable to assume that the tensile rupture will occur near the perimeter of the block, in which case the maximum tensile force  $T$  that can be carried in the plane of the membrane is:

$$T = 4s \sigma_t \cdot t \quad (5)$$

The vertical component of the tensile force must equal the block weight at equilibrium. There is a geometric relationship between the block's weight and the tensile force in the liner. By estimating the block weight and knowing the maximum allowable tensile force in the liner, the minimum angle  $\theta$  can be determined

$$e = \arcsin(w'/T) \quad (6)$$

This angle will define the minimum vertical block displacement needed to ensure that the vertical component of  $T$  is equal to the block weight  $W$ . The vertical block displacement at equilibrium is:

$$d = T \tan \theta \quad (7)$$

Based on the model shown in Figure 6, at the moment of tensile rupture the following relationship must hold true.

$$\sigma_t \cdot s \cdot t \sin \theta = \sigma_a (s + 2x)w_b \quad (8)$$

It is useful to note that the greater the angle  $\theta$  or for larger displacements and liner elongation, the greater the capacity. However, there is a limit to the allowable displacement that is governed by the elongation capacity of the liner. In this model, the following relation must not be violated.

$$\sqrt{x^2 + d^2} < (1 + e)x \quad (9)$$

Where  $e$  is the elongation at peak strength for a given liner product determined from laboratory tests. A typical value for  $e$  might be 0.2.

### 3.7 Discussion

The two models shown in Figure 6 and Figure 7 illustrate the interaction between adhesive and tensile properties of a liner. Equations 3 to 9 can be used to predict the block height that can be supported by a liner before it ruptures. In tension at a given block displacement. In Figure 7, the block height and width (square cross-section) are plotted against the vertical block displacement at equilibrium. The assumed polyurethane liner properties in Table 1 were used.

For all data plotted in Figure 7, the maximum liner elongation was less than 10%. Based on this simple model and assumed liner strengths it appears that elongation capability greater than roughly 10% may not be needed. However, if the liner's tensile strength is close to the adhesive strength, then it is advantageous to have a higher elongation capacity.

The curves plotted in Figure 7 implicitly assume a perfectly plastic material response for the liner at a target stress equal to the liner's tensile strength. Larger block displacements at equilibrium may occur if the liner has a strain-hardening response.

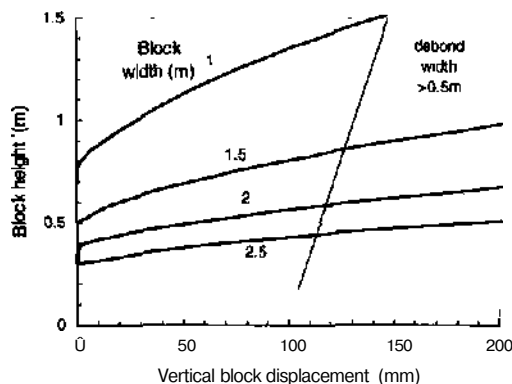


Figure 7 Block height vs. vertical block, displacement at equilibrium for a thin liner with  $\sigma_a=1\text{MPa}$ ,  $w_i=5\text{mm}$ ,  $\sigma_i=8\text{MPa}$ ,  $t=4\text{mm}$ , rock density= $2.6\text{g/cc}$ . Points plotting beyond the vertical line indicate situations where the debonded width exceeds 0.5m.

The block heights shown in Figure 7 at block displacements of zero refer to blocks that do not cause progressive adhesive debonding and hence the block is stabilized at very small displacements by the shear and/or diagonal tensile rupture support mechanisms shown in Figure 5. For example, the force required to initiate adhesive failure is  $5\text{kN/m}$ , which for a  $1\text{m}$  by  $1\text{m}$  block is equivalent to a  $0.78\text{m}$  block height. At block heights below  $0.78\text{m}$ , the model shown in Figure 5 applies and liner failure will be unlikely if the tensile/shear strength is higher than the adhesive strength. For heights greater than  $0.78\text{m}$  the model shown in Figure 6 applies, but only up to a point. Because once debonding initiates, the debonded width can quickly exceed  $0.5\text{m}$  with further increase in block height or weight. The suspension support function illustrated in Figure 6 is quite sensitive to the debonded width. In practice, effective support from a liner is probably lost once the debonded width exceeds about  $0.5\text{m}$ .

It is the combination of adhesive and tensile strengths that fundamentally controls the load the liner can support. Higher adhesive strength shifts the liner support function toward failure modes of shear or diagonal tensile rupture (Figure 5) and hence results in smaller displacements at failure. The adhesive strength to the rock must be about the same as the liner's tensile strength to most effectively utilize the tensile strength of a liner.

The tensile stresses in a liner can only counteract gravitational forces in loose rock blocks when the liner is either applied or is deformed such that a component of the tensile forces act vertically. The liner in the model presented in Figure 6 starts in a horizontal orientation and hence has no support capacity until vertical block displacement changes the liner orientation around the perimeter of the block. In a real excavation, perfectly flat backs are unusual

and hence non-horizontal sections of liner may be oriented to more effectively hold loose rock.

The issue of liner failure modes and the corresponding displacements has important implications for support design and further studies should be conducted to verify the actual failure modes under field conditions. For now, based on pull tests and limited field observations, it appears that liner failure begins as adhesive bond loss near the displaced rock. As displacements continue, the zone of adhesion failure propagates away from the block and tensile stress builds in the liner. Ultimate liner failure occurs as tensile rupture and/or adhesion loss at larger block displacements.

### 3.8 Adhesion and effective bond width

Increasing the adhesive strength and/or the effective bond width increases the load capacity of the membrane. At present, the effective adhesive bond width for most liner materials is unknown although it may be back calculated from laboratory tests. For example, two punching tests (Archibald et al. 1993) performed on three concrete blocks coated with Mine-guard may be used to give a rough estimate of the effective bond width for a polyurethane liner. The setup for the testing was similar to the conditions shown in Figure 4. The force required to displace the centre block relative to the two side blocks was  $1.73$  and  $1.10\text{kN}$ . The centre block was  $180\text{mm}$  long and adhesion was mobilized on each side of the block. Therefore the effective bond width  $w_h$  assuming an adhesive strength of  $0.9\text{MPa}$  is found from:

$$w_h = \frac{\text{load}}{\sigma_a \cdot L} = \frac{\text{load}}{0.9\text{MPa} \cdot 2(0.18\text{m})} \quad (10)$$

The estimated bond widths are  $5.3$  and  $3.4\text{mm}$ . It is quite likely that the effective bond width varies depending on the polymer type, applied liner thickness, and substrate conditions. Thicker and stiffer membranes probably have larger bond widths. For comparison, work by Fernandez-Delgado et al. (1979) and Hahn and Holmgren (1979) suggest that the effective bond width between rock and shotcrete for good adhesion is in the order of  $50\text{mm}$ .

The load capacity calculated on the basis of adhesion is probably also a function of liner thickness because thickness probably affects the effective bond width. However, the lack of tests precludes assessment of this effect. Simple laboratory testing techniques similar to those presented by Tannant et al. (1999) are needed to better quantify liner material properties.

Plated rock bolts installed after a liner is applied can function to increase the effective bond width or the adhesion and hence help mobilize the full tensile capacity of the liner at smaller block displacements.

#### 4 ECCENTRIC AND CANTILEVER LOADING

The models presented in the previous sections assume simple uniform loading conditions on the membrane. There are situations that violate this assumption. Tannant et al. (1999) described two case histories where liner failure occurred as a result of progressive tearing caused by large slabs of rock that rotated and cantilevered from the back.

In one case, a Mineguard liner was used to support a narrow drift (2m span) in highly stressed rock. A problem occurred near the advancing face where loose fractured rock caused the liner to sag between two bolts; this material was easily knocked down by a scoop bucket. A key factor contributing to the problem was the fact that the liner was not continuous to the drift face, i.e., the back at a distance of one round from the face was only supported on three sides by the liner because the newly excavated round had not been coated yet. The lack of a liner allowed roof displacements (sagging) to initiate near the edge of the blast-damaged liner and propagate away from the drift face. This created a cantilever effect in terms of the loads imposed on the liner. One positive aspect was that the liner gave ample visual warning that excessive displacements had occurred.

The other case history involved application of Mineguard to two rounds in a 3.0 to 4.3 m span drift that was driven along a swarm of sub-parallel, steeply dipping veins. The drift was excavated as part of a drift-and-fill mining method for the narrow veins and it was the third cut in a bottom-up mining sequence. While washing and scaling the roof after blasting the second round it was evident that stress-induced fracturing was occurring from the sound of "rock noises" in the roof and shoulders of the drift. The stability of the roof decreased over time due to the progressive nature of the creation of stress-induced fractures. The rock fracturing led to a fall of a large slab of rock located in the roof of the second round, which had not yet been totally coated with the liner (Figure 8). In total, roughly two tonnes of rock fell from the roof. The fall of ground also peeled some of the liner from the back.

The stress fractured slabs were observed to extend over both rounds because, coincident with the fall of ground, the roof above the first round suddenly moved downward about 50mm. However, the presence of rock bolts prevented the slabs from ultimately falling to the floor in the first round. This example shows that the addition of rock bolts may be required in many situations where a spray-on liner is used. In particular, rock bolts are probably needed in addition to a thin liner where the drift span exceeds 3 to 4m or where the rock mass quality is less than "good". When installing rock bolts the use of an automated rock bolt machine is recommended.

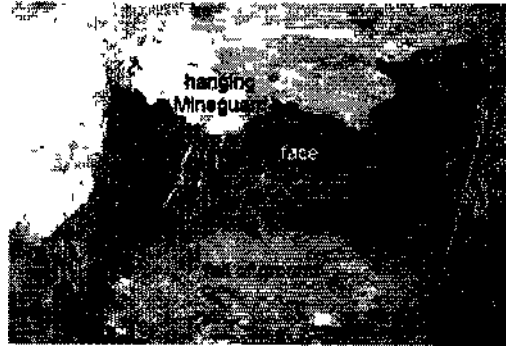


Figure 8 View towards face of a drift showing a fall of ground related to stress-induced fracturing and incomplete coverage of the roof with the liner

The two documented falls of ground occurred where the liner was not present up to the face of the drift because a new round had been previously excavated. In both cases, slabs of rock in the roof were able to move downward near the face because they were unsupported thus forming a cantilevered slab. Ultimately the slabs of rock tore through the liner at some distance from the face. In these cases the loading on the liner was concentrated at the liner's edge. High tensile stresses caused by the rotational displacement of the slab progressively ripped the liner until the slab was able to fall.

Full areal coverage by the liner is needed to minimize local straining and progressive tearing mechanisms. A continuous membrane that is firmly adhered to the rock creates effective support. Adequate rock scaling and cleaning are essential for good adhesion. Smooth liner overlap between rounds and adequate liner thickness (to bridge all rock fractures/joints) are essential for creating a continuous membrane. The use of a robotic spray arm removes the risk of ground falls while building up a continuous liner over the roof and upper parts of the walls.

#### 5 OTHER CONSIDERATIONS

Thin polymer liners have a number of attributes that warrant special attention when evaluating or designing a liner for a mining application.

##### 5.1 Timing of liner application

To gain maximum benefit from thin liner support it is important to apply the liner as soon as possible in newly blasted headings. The objective is to minimize rock mass loosening in a proactive manner. Thin liners are not very effective at 'tightening up' a rock mass that has been allowed to loosen.



The ability to rapidly apply a thin spray-on liner near the face of a newly blasted heading permits installation of support before the rock mass has time to loosen. In fact, a thin liner can be sprayed on the back of a fresh round before the blasted muck pile is removed. The rapid application rates achievable with thin liners means this type of support can be applied sooner to the rock than any other support type currently in use.

### 5.2 *Rock-support interaction*

Thin liners or shotcrete are superior to wire mesh in terms of their ability to have intimate contact with the rock and to mobilize rock interactions at small deformations. Mesh is largely a passive support that often only carries the dead weight of small rocks that have fallen between rock bolts. While a thin liner can perform a similar holding function to mesh, it is better suited to mobilizing rock-support interactions at small displacements (millimetres to centimetres). In contrast to some forms of shotcrete, the compliant nature of liners allows them to continue to function over a wider displacement range. One interesting application is the use of a combination of mesh and sprayed-on liner. Although this support is fairly expensive, it offers superior strength and deformation properties.

### 5.3 *Continuous coverage versus web*

Experience to date suggests that creation of an unbroken thin membrane over the entire rock surface provides the best support. The liner follows the contour of the rock surface although the liner thickness is usually greater near open fractures or sharp concave depressions in the surface.

Intelligent spraying equipment may be developed to identify the location of joints and fractures and then only spray the liner material in these locations thus creating a liner that functions like a spider's web. This could significantly reduce polymer material consumption and decrease application times. A web of liner material bridging across the joints and fractures would help prevent relative rock displacements. However, if ground conditions become so severe that the polymer web cannot prevent these displacements it is likely that a liner consisting of a continuous membrane would perform better. A continuous liner is simply more robust and has higher load carrying capacity than a web of polymer material. Nevertheless, where ground conditions are appropriate, a web of polymer material is likely to be very effective and very economical.

### 5.4 *Performance near blasts*

Field observations have shown that a thin liner performs well in close proximity to blasts. When mesh

and bolts are used within a metre of an advancing heading it is usual to see extensive damage to the mesh after each blast. The mesh is torn by the fly rock. A typical drift round will damage the mesh over a distance of roughly 3 to 5m from the blasthole collars. When supporting the back in preparation for the next round, some of the damaged mesh must be removed and new mesh installed. This is a time consuming process that exposes personnel to hazards from small falling rocks as well as cuts from torn mesh.

Thin sprayed-on liners can be used up to the face, and on the face itself if needed. After a round is blasted, portions of the liner further than one metre from the face typically sustain only minor damage such as small nicks, cuts and abrasions. The damage is most pronounced where the supported surface protrudes into the drift or on surfaces that face the blast. As expected, the worst damage occurs immediately adjacent to the face. At these locations, the liner can be peeled back about 0.3 to 0.5m from the face by the blast. Further from the face, the liner typically experiences only small nicks and cuts.

Near the face, where the liner has been torn from the rock there will normally be flaps of liner material adhering to the rock. These flaps of material must be cut away in preparation for the next application of the liner. In small headings, the flaps of liner material pose a problem. For example, when mucking out a drift, a scoop can accidentally catch a flap of liner and pull off quite a large section of the still good liner. Further equipment development is needed to simplify the process of trimming away flaps of liner material created by the blast. One operational procedure that has minimized this problem is to taper the thickness of the liner toward the face. The thinner liner near the face is more likely to tear without peeling off the rock. This leaves a narrow zone where the liner is destroyed by the blast but there is a clean transition to essentially intact liner.

### 5.5 *Long-term performance*

Polymeric liners have not been used for more than a few years in routine mining applications. Therefore, little operational evidence exists concerning their longevity in a mining environment. Initial research suggests that most materials in use today have very good resistance to acids and bases (Archibald & DeGagne 2000). Some polyurethane liner materials alter their colour and appear to degrade when exposed to sunlight. This should not be a concern in the underground environment.

One concern with polymer liners is their creep characteristics. Simple tests have demonstrated that most liner materials will creep and rupture at stresses much less than the values quoted for their tensile strengths. The impact of creep on the load capacity of a liner in conditions where a liner is sup-

porting the gravitation load from loose broken rock is unknown. Further research is needed to evaluate the performance of polymer liners under sustained loading conditions. The Canada Centre for Mineral and Energy Technology CANMET is working with Falconbridge to address this issue. Fortunately most liner materials can sustain large strains prior to rupturing. This allows for visual identification of areas experiencing problems and allows remedial actions to be taken before a fall of ground occurs.

### 5.6 *Safely*

Mine accident statistics demonstrate that the activities associated with the installation of mesh are relatively hazardous. Mesh installation is a labour intensive and manual operation and personnel are exposed to small rock falls, cuts, slips and strains in the process. In contrast, liner spraying is amenable to robotic application, which essentially eliminates these hazards.

### 5.7 *Rock visibility*

Thin liners can be sprayed on the face of an advancing drift to provide support against hazards such as small rockbursts. One advantage of thin liners compared to shotcrete is the ability to still see major rock structure and bootlegs after application of the support. The bootlegs from previous blastholes can be easily identified such that the new blastholes are not collared near the bootlegs.

It is also easy to identify features such as joints and rock type where the rock 'roughness' varies from one type to another.

Liner materials that are white in colour provide a major improvement in the general lighting conditions in an underground environment.

### 5.8 *Dirty or weak, crumbly rock*

Thin liners have not been used successfully on weak, crumbly rock. Where the rock is weak or covered in dust it is impossible to create good adhesion between the liner and the rock. Without good adhesion a liner does not work. There have been cases where small pockets of high-grade sulphide ore have coated by liners. High-grade sulphide ore can have a sugary, crumbly texture and little tensile strength. As expected the liner did not adhere to this rock type. The rock itself must possess sufficient tensile strength,

### 5.9 *Contamination of ore*

When mining through supported areas, the ground support becomes mixed with the blasted ore. Some studies have indicated that the presence of shotcrete in ore may cause detrimental effects in the milling

and mineral recovery process. It is not known if this is an issue with the various types of liner materials. Fortunately, the quantity of liner material needed to support a given area will be substantially less than for shotcrete.

### 5.10 *Application rates*

INCO completed costing and time studies for the activities needed to install various support types (Espley-Boudreau 1999). The studies were based on a 4.9m by 4.9m drift with a 3.7m drilled round achieving 3m advances. It was assumed that bolts and mesh were installed using a scissor-lift truck with hand-held stopper and jack-leg drills. The shotcrete was applied manually with dry-mix equipment, and the polymer liner was sprayed with a hand-operated spray gun. The application rates were found to be 0.11 to 0.15m<sup>2</sup>/min for 1.8m long mechanical rock bolts and welded-wire mesh; 0.1 to 0.33m<sup>2</sup>/min for manual application of 50mm thick fibre-reinforced shotcrete (no rock bolts); and 1.8 to 2.3m<sup>2</sup>/min for polymer liners (no rock bolts). When the labour component is included, the study found that polymer liners can be applied at a rate of about 60m<sup>2</sup>/man-shift versus 20m<sup>2</sup> or 40m<sup>2</sup> per man-shift for bolts and mesh or shotcrete respectively.

The application rate for shotcrete can be increased by an order of magnitude by adopting the wet-mix method and using remote semi-automated equipment. Similar productivity improvements are expected for thin spray-on liners once they become more widely used and specialized spray equipment is developed.

### 5.11 *Costs*

The material costs for some polymer liner materials are presently quite high, ranging between Cdn \$25/m<sup>2</sup> and \$50/m<sup>2</sup> for an assumed application thickness of 4mm. These costs on a per metre basis are similar to 50mm of steel fibre reinforced shotcrete. The material costs for rock bolts and mesh cost the least, at about \$10 to \$13/m<sup>2</sup>. However, it must be recognized that the installation of conventional mesh and bolts is both time consuming and labour intensive. In all cases, the total support costs involving labour and equipment were much larger than the material cost. It is important to remember that the material costs do not control the overall economics of the support selection.

The economic benefits from using thin sprayed polymer liners are realized by the higher productivity created by reduction of the time needed for support installation. Further gains are possible when material transportation and handling cost are considered. Compared to shotcrete, a lot less material needs to be moved underground to the working face when using thin polymer liners.

Studies by INCO indicate that the total support cost using polyurethane liners (roughly \$125/nr) is similar to bolts and mesh and cheaper than mesh-reinforced shotcrete (Espley-Boudreau 1999). This cost does not account for substantial productivity improvements that are forecast to occur once semi-automated spraying of thin liners is implemented.

#### 6 AN EXAMPLE OF CURRENT THIN LINER USE IN CANADIAN MINES

Falconbridge Ltd. is making routine use of TekFlex as mesh replacement in cut and fill stopes and permanent development drifts at the Fraser Mine (Figure 9). Thin liners combined with systematic mechanized rock bolting are being used to help support the back of 80 to 85% of all headings. Mesh is still used in the areas that experience stress-driven fracturing resulting in the generation of slabby rock (Pntchard per. coram. 2001) or in areas where the surface roughness of the rock is high thus requiring excessive quantities of liner per metre of drift (Pntchard et al. 2001).

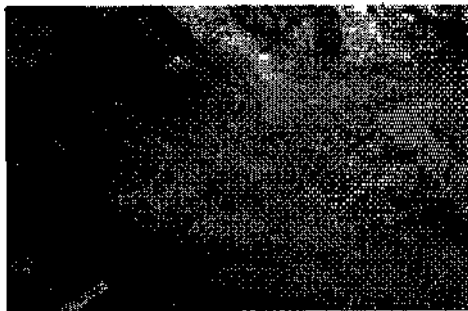


Figure 9 TekFlex spraying at Fraser Mine

Three key factors contributing to successful implementation of thin liners at Fraser Mine were (1) adopting stricter blasting and site preparations procedures, (2) better use of mechanized bolters, and (3) improved materials handling capabilities (Pntchard et al. 1999). One challenge that was overcome was the implementation of strict quality control procedures for site preparation, which included washing and scaling the back. When operators failed to follow the site preparation procedures, poor adhesion of the liner to the back resulted.

Perimeter control blasting techniques involving improved attention to blasthole location and alignment were found necessary to produce good quality ground conditions for a liner application. Once the operators realized the importance of a smooth, sound, clean rock surface, better care was devoted to the drilling of blastholes.

The support installation procedure is as follows. Once a heading is mucked out, the area is mechani-

cally scaled to bring down large loose material. The round is then bolted with a mechanical bolter. Before spraying begins high pressure water scaling is performed. Once scaling is completed, the liner materials are mixed and readied for application. A calculated quantity of the liner material is sprayed onto the surface. A total of 200 litres is sprayed in a 4.6m wide by 3.5m high drift (4.2m advance), and down the walls to a height of 2.5m from the floor. About 300 litres is sprayed in a cut and fill environment where the breasting width is 11m, with 4.2m advance. TekFlex is sprayed with a mobile boom arm to cover the back to a thickness of about 4mm, and the walls to a thickness of 3mm.

One benefit from adopting a thin liner was increased awareness of the importance of high quality work. This resulted in improved perimeter control, drill hole alignment, and a reduction in bootlegs. Furthermore, rock bolts are now being placed where they are needed most to support the rock rather than restricted to specific locations in order to hold sheets of mesh in place. When mesh is used, the bolting pattern is eight bolts per (1.8m x 3.4m) sheet of welded-wire mesh on a 3-2-3 pattern resulting in a bolt density of about 1.6 bolts/m<sup>2</sup>, accounting for mesh overlap. Use of mesh often results in the installation of more bolts than are required for the ground conditions. Elimination of the mesh has allowed a wider bolt spacing (1 bolts/m<sup>2</sup>) thus nearly doubling bolter productivity. The use of a thin liner enables the bolting density to match the ground conditions.

#### 7 CONCLUSIONS

Thin liner support is an emerging technology that is applicable to underground support of blocky rock masses. A variety of different liner materials are currently being investigated and some are now being used in routine support applications within Canadian mines.

Thin polymer liners have performance characteristics that lie between those of shotcrete and mesh. They are a welcome addition to the 'tool box' of support types and have a role to play where rapid application rates and areal support of rock are needed.

The liner must adhere well to the rock and hence, the use of thin spray-on liners is not recommended where the rock surfaces are dirty or can not be cleaned or where the rock has a crumbly texture. A continuous liner that is firmly adhered to the rock creates effective rock support.

Various approaches were used to examine the load capacity of a liner. Interpretation of the available test data and introduction of simple support models show that the two likely failure modes are adhesion loss and tensile or shear rupture of the membrane. Different failure modes occur depending

on the relative tensile and adhesive strengths of the liner and the anticipated magnitude of the rock displacements.

Virtual all liner materials are very effective at holding in-place small pieces of rock. When a liner is used to support a larger area it can do so through a combination of adhesion and shear strength at small (<1mm) relative block displacements. If excavation conditions generate larger block displacements, the liner acts like a supporting membrane and a combination of adhesive strength, tensile strength, and liner elongation serve to eventually create force equilibrium in a displaced loose block.

More research is needed to determine design values for the tensile, shear, and adhesive strengths of different liner materials. It is equally important to gain a better understanding of the effective bond widths that carry adhesive stress during progressive debonding of a liner from the substrate material. Field trials are useful for identifying liner performance and potential failure modes and aid with the development of reasonable models for liner design.

There are a wide variety of factors that must be considered before adopting wide spread use of thin sprayed on liners. Fortunately, it appears in some cases that liners offer increased safety, better productivity, and lower overall mining costs compared to conventional bolt and mesh or shotcrete.

#### ACKNOWLEDGEMENTS

The numerous field and laboratories tests performed by the author while a member of the Geomechanics Research Centre were supported by INCO Ltd., Falconbridge Ltd., FosRoc, and Master Builders. Contributions from S. Espley, C. Pritchard, other members of the Geomechanics Research Centre, J. Archibald, D. Degville, and A. Spearing are gratefully acknowledged.

#### REFERENCES

Archibald J.F. & DeCagne D.O. 2000. Recent Canadian advances in the application of spray-on polymeric linings. Presentation 10 the Mining Health and Safety Conference, Sudbury, 20p.

Archibald J.F. & Lausch P. 1999. Thin spray-on linings for rock failure stabitizationo *Proc. 37th US Rock Mechanics Symp.*, Vail. *Rock Mechanics for Industry*, Balkema, Rotterdam, 617-624.

Archibald J.F. Espley S.J. St. Lausch P. 1997. Field and laboratory response of Mineguard spray-on polyurethane liners. *Proc. Int. Symp. on Rock Support*, Lillehammer, 475-490.

Archibald J.F. Mercer R.A. & Lausch P. 1992. The evaluation of thin polyurethane surface coatings as an effective means of ground control. *Proc. Int. Symp on Rock Support - Rock Support in Mining and Underground Construction*, Balkema, Rotterdam, 105-115.

Archibald J.F. Mercer R.A. & Lausch P. 1993. Innovative ground support using rapid deployment of spray-on liners *Canadian Institute of Mining and Metallurgy Annual General Meeting*, Alberta.

Espley S.J. Langille C. & McCreath D.R. 1995. Innovative liner support trials in underground hardrock mines at INCO Limited. *Canadian Institute of Mining and Metallurgy Bulletin* 88(992<sup>o</sup>),66-74.

Espley S.J. O'Donnell J.D.P. Thibodeau D. & Paradis-Sokoloski P. 1996. Investigation into the replacement of conventional support with spray-on liners. *Canadian Institute of Mining and Metallurgy Bulletin*, 89(1001 ), 135-143.

Espley S.J. Tannant D.D. Baiden G. & Kaiser P.K. 1999. Design criteria for thin spray-on membrane support for underground hardrock mining. *Canadian Institute of Mining and Metallurgy Annual General Meeting*, Calgary, published on CD-ROM, 7 p

Espley-Boudreau S.J. 1999. Thin Spray-on Membrane Support and Implementation in the Hardrock Mining Industry. M.A.Sc. thesis, Laurentian University, Sudbury, Ontario, 31 lp.

Femandez-Delgado G., Cording E.J., Mahar J.W., & Van Sint Jan M.L. 1979. Thin shotcrete linings in loosening rock. *Proc. 1979 Rapid Excavation and Tunneling Conference*. American Institute of Mining, Metallurgical, & Petroleum Engineers, 790-813.

Hahn T. & Holmgren *i*. 1979. Adhesion of shotcrete to various types of rock surfaces. *Proc. 4<sup>th</sup> Congress of the Int. Society for Rock Mechanics*. Balkema, 431-439.

Hoek E. & Brown E.T. 1980. *Underground Excavations in Rock*. Institute on Mining and Metallurgy, London, England.

Kaiser P.K. & Tannant D.D. 1997. Use of shotcrete to control rockmass failure. *Proc. Symp. on Rock Support*, Lillehammer, 580-595.

O'Donnell J.D.P. & Tannant D.D. 1998. Field pull tests to measure insitu capacity of shotcrete. *Canadian Institute of Mining and Metallurgy Annual General Meeting*, Montreal, published on CD-ROM.

Pritchard C. Swan G. Henderson A. Tannant D. & Degville D. 1999. TekFlex as a spray-on screen replacement in an underground hard rock mine. *Canadian Institute of Mining and Metallurgy Annual General Meeting*, Calgary, published on CD-ROM, 6 p.

Pritchard C. Swan G. Hill R. & Amick M. 2001. TekFlex as a spray-on screen replacement in an underground hard rock mine. *Mining Engineering, SME* in press, 4p.

Spearing A.J.S. & Champa J. 2000. The design, testing and application of ground support membranes for use in underground mines. *Proc. MASSMIN2000*, AusIMM.

Tannant D.D. 1995. Load capacity and stiffness of welded-wire mesh *Proc. 48th Canadian Geotechnical Conference*, Vancouver. 729-736.

Tannant D.D. 1997. Large-Scale Pull Tests on Mineguard™ Spray-on Liner. Report to INCO Limited, Copper Cliff.

Tannant D.D. & Kaiser P.K. 1997. Evaluation of shotcrete and mesh behaviour under large imposed deformations. *Proc. Symp. on Rock Support*, Norway, 782-792.

Tannant D.D., Kaiser P.K. & Maloney, S. 1997. Load-displacement properties of welded-wire, chain-link, and expanded metal mesh. *Proc. Symp. on Rock Support*, Norway, 651-659.

Tannant D.D. Espley S. Barclay R. & Diederichs M. 1999. Field trials of a thin sprayed-on membrane for drift support. *Proc. P<sup>th</sup> ISRM Congress on Rock Mechanics*, Balkema, 1471-1474.

Tannant D.D. Swan G. Espley S. & Graham C. 1999. Laboratory test procedures for validating the use of thin sprayed-on liners for mesh replacement. *Canadian Institute of Mining and Metallurgy Annual General Meeting*, Calgary, published on CD-ROM, 8 p.

Wojno L. & Kuijper J.S. 1997. Spray-on, user friendly and flexible support for mine excavations. *Proc. Int. Symp. on Rock Support*, Lillehammer, 671-683.

## A Case Study on Safety Factor and Failure Probability of Rock Slopes

M.U.Ozbay

Colorado School of Mines, Mining Engineering Department, Golden, Colorado, USA

**ABSTRACT:** Performances of two different statistical methods in determining failure probability of rock slopes are evaluated using data from insitu plane failure cases. The first method, called "direct method", defines cohesion and friction angle as normal distributions and calculates a failure probability from the safety factor distribution. The second method, called "maximum likelihood method" estimates unique values for cohesion and friction angle by maximizing the safety factor distribution. The analyses using data from 48 plane failure cases show that both methods predict similar mean values for cohesion and friction angle, and thus, for safety factor. However, the maximum likelihood method predicts less scatter in the safety factor distribution, thus lower failure probabilities than the direct method for a given slope geometry.

### 1 INTRODUCTION

This paper describes the applications of the two different statistical methods to estimate probability of failure of slopes excavated in rock masses. Both methods use data from failed cases and determine failure probability from safety factor distributions. The "direct method" is based on the assumption that the safety factor  $S$  of a slope at failure is equal to 1.0 and back-calculates the strength parameters, namely the cohesion and friction angle, using the geometrical data obtained from failed slope cases (Hoek and Bray 1981). Depending on the level of inaccuracies involved in the geometrical measurements, the strength parameters obtained from the failed cases are most likely to differ from each other. These strength parameters, namely cohesion and friction angle, can be set as statistical distributions and used in the safety factor formula to determine a safety factor distribution. This safety factor distribution can then be used to obtain failure probabilities of slopes with different geometries. The "maximum likelihood method", also accepting that the determination of the true values of the failure geometries is practically impossible, assigns a particular statistical distribution to the safety factors from the failed cases and back-calculates the values of cohesion and friction angle that by maximizing this distribution around the central value  $S=1$  (Salamon (1999)). This process produces a standard deviation for the maximized distribution and unique values for cohesion and friction angle. The values of cohesion and friction an-

gle now can be used to design a new slope and the failure probability of this slope can then be calculated from the maximized safety factor probability distribution.

In die following, these two methods are described in further detail and evaluated using a data set established from 48 plane failure cases that occurred in the benches of a large open pit mine.

### 2 IN-SITUDATA

The data used is from 48 dry plane failure cases that occurred along a major joint set in the benches of a large open pit mine (Calderon, 2000). It contains only those collected from clearly defined and well-exposed plane failures. Each case is described in detail and includes the parameters relating to geometry of the failed blocks and joint inclination angle. Table 1 gives the range of the parameters measured during the surveys.

The shear strength of the joint planes is assumed to be governed by the Mohr-Coulomb shear failure criterion given as

$$\tau = c + \sigma_n \tan \phi \quad (D)$$

Table 1 Range of geometrical parameters measured for the failed cases.

Dip of slip plane (°)	37-66
Dip of the slope face (°)	57-85
Height of the sliding block (m)	10.8-32.0
Weight of the sliding block (kN)	390.0-11100

where  $c$  = cohesion,  $\sigma_n$  = normal stress acting on the joint surface, and  $\phi$  = joint's friction angle. The failure is assumed to occur according to the limiting equilibrium condition along dry discontinuity planes with no tension crack, which is expressed as

$$S = \frac{cA + W \cos \psi_p \tan \phi}{W \sin \psi_p} \quad (2)$$

where  $S$  = safety factor,  $W$  = weight of the sliding block, and  $\psi_p$  = the inclination angle of the failure plane.

### 3 DIRECT METHOD

In the direct method, it is assumed that the failure occurs when  $S=1$ , which, when substituted in (2), gives

$$c = \frac{W \sin \psi_p - W \cos \psi_p \tan \phi}{A} \quad (3)$$

If the geometry and the rock mass density from any two failed cases are known, Eq. (3) can be set for these cases and solved simultaneously to calculate the values of cohesion and friction angle. In reality, true measurement of the blocks and rock mass density is practically impossible. However, if there are several cases of failures and their geometries are measured with humanly possible accuracy, the statistical distributions of the cohesion and friction angle values can be established from  $n(n-1)/2$  solutions;  $n$  being the number of failed cases.

Figure 1 shows the cohesion and friction angle values obtained from 48 plane failure cases. The solutions resulted in 948 points of intersection in the positive quadrant. The mean and standard deviation of these cohesion and friction angle values corresponding to the points of intersections are given in Table 2.

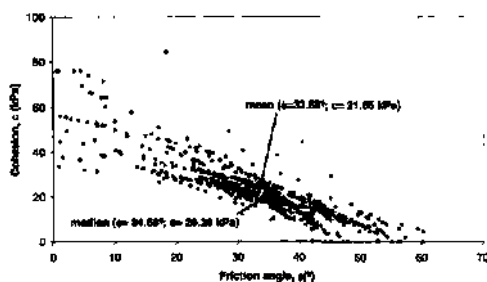


Figure 1. Cohesion and friction angle values calculated from 48 failed cases.

Table 2. Statistical parameters for cohesion and friction angle obtained from 48 failed cases using the direct method.

	Cohesion ft(Pa)	Friction Angle D
Mean	21.7	33.9
Standard deviation	12.50	9.84
Median	20.3	34.7

Based on the mean and standard deviation values given Table 2, a normal distribution of 10000 cohesion and friction angle values generated using Monte-Carlo technique is shown in Figure 2. Both data sets have large standard deviations, which result in negative values in the data sets, which are truncated to exclude negative values but retaining similar statistical parameters. The statistical parameters for the truncated normally distributed data sets are given in Table 3, which shows that the mean and standard deviation of the new data sets are sufficiently close to the original data given in Table 2.

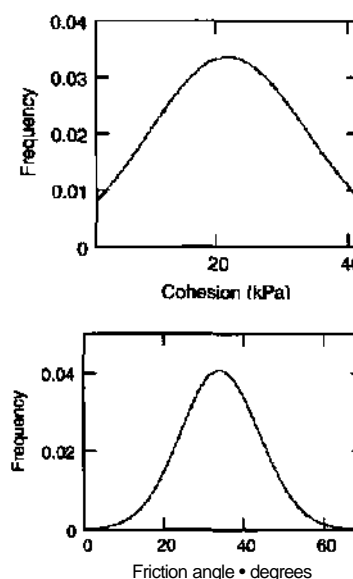


Figure 2. The normal distributions for cohesion (top) and friction angle (bottom) generated using the statistical parameters given in Table 2.

Table 3. Statistical parameters of cohesion and friction angle data from generated and truncated normal distributions.

	Mean	Median	St. dev.
Cohesion (Figure 2) kPa	21.7	21.8	11.9
Friction angle (Figure 2) $\phi$ (°)	33.9	34.0	9.8
Cohesion (truncated) c (kPa)	21.9	21.8	11.5
Friction angle (truncated) $\phi$	33.8	34.0	9.8

The truncated frequency distributions of cohesion and friction angle are used for determining the safety factor distributions. For the slope geometry parameters, the mean values from the 48 cases are used, and these are given in Table 4.

Table 4 The geometrical parameters used for determining the safety factor distribution.

	Mean	St. dev
Failure plane angle $\nu_p$ (°)	50.9	7.4
Block base areas A (m <sup>2</sup> )	25.9	7.3
Block height H (m)	16.4	4.6
Slope face angle $\theta$ (°)	67	51

The safety factors calculated using the strength parameters in form truncated distributions in Table 3 and the geometrical parameters given in Table 4 are shown as a histogram in Figure 3. The mean and standard deviations of the safety factor population are 1.001 and 0.324, respectively, and this, as a normal distribution, is also given in this figure.

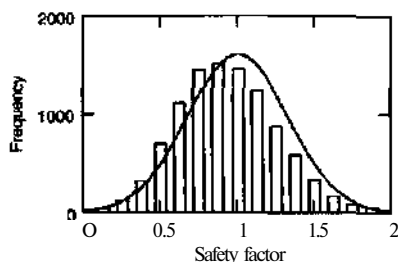


Figure 3. The safety factor distribution as obtained from the direct method

The validity of the mean values of the cohesion and friction values given in Table 3 can be assessed by plotting the joint's shear strength against the acting shear stress for the 48 failed cases, as shown in Figure 4. The straight line in this plot marks the location of shear stress = shear strength, i.e.  $S=1$ . Most values of calculated safety factors lie close to this line, indicating that the method produced results that fit the insitu data reasonably well.

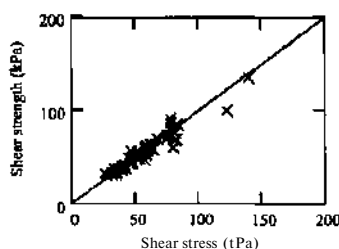


Figure 4. The safety factors for the 48 failed cases calculated using the cohesion and friction angle values estimated from the direct method.

#### 4 MAXIMUM LIKELIHOOD METHOD

In this method, the safety factor distribution function is maximized, with the objective of having the cohesion and friction values to result in a safety factor population concentrated around the central value of  $S=1$ . This can be achieved by multiplying the ordinates of the frequency distribution function corresponding to each failed case, that is maximizing the function

$$M(c, \phi, \sigma) = \prod_{i=1}^n f[\ln S(c, \phi, \sigma)] \quad (4)$$

where  $\sigma$  is the standard deviation  $S$ , is the safety factor formula as given in (2) above and thus  $f(\cdot)$  represents lognormal distribution function for assumed for the safety factors from the 48 failed cases.

The cohesion and friction values calculated from the maximum likelihood method, using the geometrical parameters given in Table 4, are given in Table 5. The standard deviation of 0.101, resulting from the safety factor population, is much smaller than that 0.324, obtained by using the direct method.

Table 5 The strength parameters obtained from the maximum likelihood method

	Mean	Median	Standard Dev
Cohesion $c$ (kPa)	19.5		
Friction angle $\phi$ (°)	35.4		
Safety factor	0.9948	0.9946	0.101

Based on the values given in Table 5, the frequency histogram and the distribution function are given in Figure 5. To evaluate the validity of the cohesion and friction values calculated from the maximum likelihood method, Figure 4 is re-plotted in Figure 6. The plot is almost identical to that of the direct method, which is expected since the mean values of cohesion and friction determined by the use of the either method are similar.

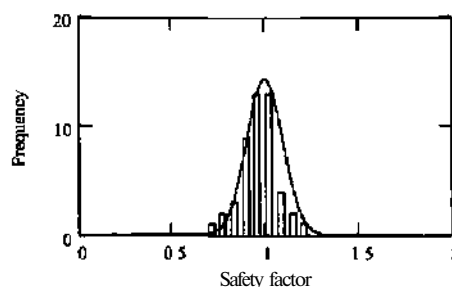


Figure 5 The safety factor distribution as obtained from the maximum likelihood method

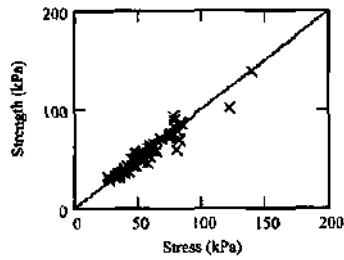


Figure 6. The safety factors for the 48 failed cases calculated using the cohesion and friction angle values estimated from the direct method.

## 5 FAILURE PROBABILITY ESTIMATIONS

The failure probability estimations are made from the cumulative density functions originated from the direct and maximum likelihood methods described above. Two different approaches are considered in estimating failure probabilities. In the *probability of failure* approach, the probability density and cumulative density functions are re-built for a particular safety factor being considered. The proportion of the cases with  $S < 1$ , determined from either of these functions, is then taken as the probability of failure. As an example, consider reducing the slope angle from the original  $67^\circ$  to  $57^\circ$ . The safety factor frequency distribution, obtained by using  $57^\circ$  for the direct method, is given in Figure 7(top). The deterministic safety factor in this case is  $S = 1.54$ , which is about the same as the mean of the distribution in this figure. The proportion of the number of cases of  $S < 1$  gives the failure probability for this particular safety factor. Alternatively, the failure probability can be read directly from the ordinate of the cumulative density function shown in Figure 7(bottom), which for this example happens to be 17%.

In the *probability of survival* approach, always the frequency distribution obtained from the failed cases (e.g. Figure 3 or Figure 5), or their cumulative density function (Figure 8), is used regardless of the value of the safety factor being considered. Continuing with the example, the safety factor resulting from reducing the slope to  $57^\circ$  is 1.54. The proportion of the cases with  $S > 1.54$  makes up about 4.7% of all cases. That is, the probability that a slope with  $S = 1.54$  being part of all the failed cases is 4.7%. In the cumulative distribution curve, this corresponds to  $1 - F(S)$ , which is called *probability of survival*, (Salamon, 1999), with  $F(S)$  being the cumulative distribution function. In the maximum likelihood method, since cohesion and friction are determined as constants, probability of failure approach described above becomes inapplicable.

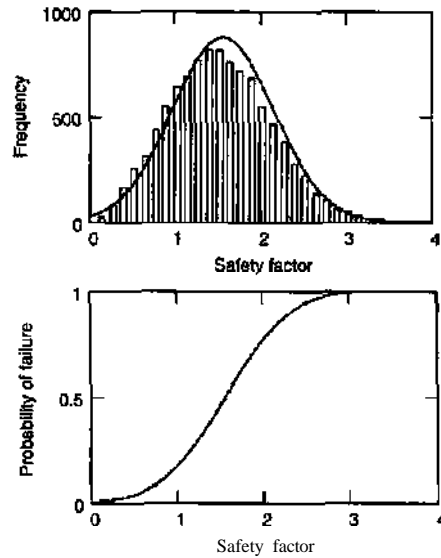


Figure 7: Frequency distribution and histogram of the safety factor distributions (top), and the cumulative density function (bottom) obtained from the direct method for  $57^\circ$  slope angle ( $S = 1.54$ ).

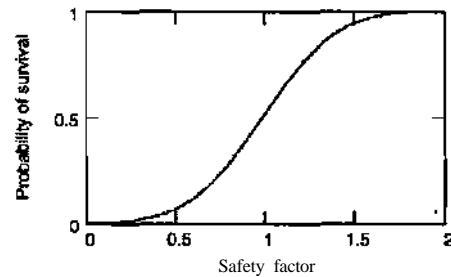


Figure 8: Cumulative density function of the probability function in Figure 3.

Table 6. Failure probabilities based on the "probability of failure" and "probability of survival" approaches calculated from the direct and maximum likelihood methods.

Slope angle	Safety factor	Failure probability %		
		Direct method	Maximum likelihood	Probability of survival
62	0.97	50.0	53.4	62.6
62	1.13	34.3	34.3	12.3
57	1.54	17.0	4.7	0.0012
52	5.70	5.2	0	0

Failure probabilities calculated using the *probability of failure* and *probability of survival* approaches are compared in. As seen, the two approaches, as well as the two methods of determining



safety factor distributions give significantly different results, especially at increased safety factor values. At  $S=5.7$ , the probability of survival predicts almost 0% failure probability, while the estimation with probability of failure approach is 5.2%, which is probably unrealistic. When comparing the methods, it can be seen that the maximum likelihood method predicts lower failure probabilities for  $S > 1$  than the direct method.

Based on the discussions above, the following comments are noteworthy:

1. The reason that maximum likelihood method gives less probability of failure is due to lower standard deviations of the safety factor distributions originally calculated with this method.
2. Use of lognormal distribution with the maximum likelihood method, by definition, does not allow negative cohesion and friction angle values, thus probably more realistic than using normal distribution with the direct method.
3. Maximum likelihood method is less affected by the values of the geometrical parameters in the safety factor formula.
4. The maximum likelihood method appears to be a viable statistical tool and the potentials offered by this method are well worthy of further trials and research.

## 6 CONCLUSIONS

- For the failed cases analyzed here, the safety factor values calculated from the direct method and the maximum likelihood methods are similar, indicating that the methods are equally applicable in deterministic design.

- The larger standard deviations in the direct method require truncation of the distribution functions to avoid negative cohesion and friction angle values in the data sets. The negative strength parameters do not result in the maximum likelihood.
- The larger standard deviations in the direct method affect the failure probability calculations. Increasing the safety factor in this method is less effective in reducing the probability of failure when compared to maximum likelihood method. This observation is significant as it points out that the degree of improvement in failure probability by increasing safety factor is dependent on the method used to determine the probability density function.
- The probability estimation using the concept of probability of survival appears to be a viable concept and deserve further research and trials.

## REFERENCES

- Salamon, M D G (1999) Strength of coal pillars from back-calculation. Proceedings of the 11<sup>th</sup> US Rock Mechanics Symposium, pp 29-36 Vail, Colorado, USA
- Calderon, A. R (2000) The application of back-analysis and numerical modeling to design of a large pushback in a deep open pit mine MS thesis. Department of Mining Engineering, Colorado School of Mines, Golden, Colorado, USA.
- Calderon, A. R. & Ozbay, M U. (2001) Shear strength determination by back-analysis of slope failures using maximum likelihood method ARMA 11<sup>th</sup> US Rock Mechanics Symposium, Washington DC, USA.
- Hoek, E. & Bray, J.W. (1981) Rock slope engineering. London; IMM.

