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Evaluation of Slope Stability by Numerical Methods

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ABSTRACT: Numerical methods are relatively recent compared to the analytical methods of slope stability analysis known as "limit equilibrium". Numerical techniques are used to obtain stress and strain distribution at any point of a defined grid. They are particularly useful for the analysis of slope stability when it is subject to various types of loading or when it has complex geometry. In this paper, we present a means of combining a numerical method using a finite difference code and circular failure analysis by the method of limit equilibrium. The state of stress is calculated by a finite difference code and then used in a program based on the limit equilibrium method for the calculation of the safety factor. We applied this new approach to a finite difference code (Itasca-FLAC^{2D}). For this purpose, we used a parameter called the global safety factor. This safety factor İs defined as the average of local safety factors, which are calculated by two methods: the method known as "surface of failure locally imposed", and the method known as "surface of failure locally optimized". A computer program integrated in FLAC makes it possible to find the critical failure surface automatically, and it displays the corresponding safety factors. We applied this development to a real case.

1 INTRODUCTION

The most widely accepted methods of analyzing slope stability problems are based on limit equilibrium and experimental data. The stability of slopes has been traditionally evaluated using a variety of two-dimensional limit equilibrium methods (Merrien-Soukatchoff& Omraci, 2000):

Simplified methods: representative of mis group is the Fellenius method (1936); it is the basic method of limit equilibrium.

A second group, to which Taylor's method (Taylor, 1937) belongs, satisfies all equilibrium requirements. Taylor's method is based on the assumption that the kinematical function represents a circular arc.

Generalized methods of slices: this group consists mainly of variations of the approach presented by Morgenstern and Price (1965), and Janbu (1954). In this group, the kinematical function is left unspecified. In order to provide a computation scheme that enables the determination of the critical slip surface, a variational reasoning has been applied to the generalized methods of slices (Revilla and Castillo, 1977). A similar type of analysis had been carried out earlier (Donnan, 1965; Garber, 1973). An improved variational formulation of the slope stability problem was presented by Baker and Garber (1977).

The numerical memods, on the other hand, are widely accepted for analyzing stress and displacements and are, therefore, seldom accepted for analyzing the safety problem. They require much longer times to make just one safety factor calculation. However, recent advances in the computational speed of personal computers now permit safety factor calculations with numerical models to be made routinely. Several attempts have been made in this area, such as the research of Merrien-Soukatchoff (2000), Korini (1999), Stanley (1996), Fry and Brunet (1999) and Itasca (1996).

In this paper, a new method based on both the finite difference method and the limit equilibrium analysis with the objective of better simulation and interpretation of slope failure is given.

2 GLOBAL FACTOR OF SAFETY

We define the global factor of safety as the average value of the local factors of safety:

$$FS\_global = f(FS_1\_local, FS_2\_local, \dots FS_n\_local) \quad (1)$$

n: number of elements on the considered surface of failure;

/' function binding the local safety factors to the global factor; for example, the average in our case.

Normally, we can adopt different forms of failure surface. In this paper, we assume a circular surface.

3 APPROACHES TO CALCULATING THE LOCAL FACTOR OF SAFETY

In order to obtain the global factor of safety, we must determine the values of local safety factors for each point (element) of the slope (model). For slopes and embankments, the factor of safety is traditionally defined as the ratio of the soil's actual shear stress to the minimum shear strength required to prevent failure.

Two approaches to compute this factor of safety with a numerical method are used.

3.1 *Surface of failure locally imposed*

In our study, we adopt a linear failure criterion like Mohr-Coulomb. This criterion is defined by two constants: the cohesion c and the friction angle (ft.

The state of stress for any zone can be expressed in terms of the principal stresses Oi and 03. This stress state, in general, could be plotted as a circle on a Mohr diagram (Fig. 1). Failure occurs if this circle touches the failure envelope. Drawing the line BE, this tangents the MoVs circle at point B. The line is also parallel to the failure line (Stanley, 1996). By geometric analysis, we obtain (Fig. 1):

$$\text{From } \Delta AFD : \tan\phi = AD / FD = \tau_a / FD \Rightarrow \tau_a = FD.\tan\phi$$

where x_a is the shear stress of the considered element.

We also have:

$$FD = c.\text{ctan}\,\phi + (CO\text{-}CD)$$

$$FD = c/\tan\varphi + 0.5\times(\sigma_1+\sigma_3) - 0.5\times(\sigma_1-\sigma_3)\times\sin\varphi$$

Then:

$$\tau_a = [c/\tan\varphi + 0.5\times(\sigma_1+\sigma_3) - 0.5\times(\sigma_1-\sigma_3)\times\sin\varphi]\ \tan\varphi \quad (1)$$

σ_1,σ_3: principal stresses are the results of the numerical model.

From ÄBCD: $\cos\phi = BD / BC = \tau_d / 0.5\times(\sigma_1-\sigma_3)$

Xa- the developed shear stress on the possible failure surface.

The value of the ratio *{%Jx&)* is defined as the local factor of safety of an element with a given stress state:

$$FS = \frac{\tau_a}{\tau_d} = \left\{\frac{[c/\tan\varphi + 0.5\times(\sigma_1+\sigma_3)}{0.5\times(\sigma_1-\sigma_3)\cos\varphi}\ \frac{-0.5\times(\sigma_1-\sigma_3)\times\sin\varphi]\tan\varphi}{0.5\times(\sigma_1-\sigma_3)\cos\varphi}\right\} \quad (2)$$

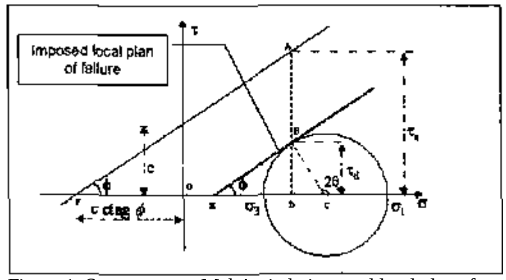

Figure 1 Stress state on Mohr's circle-imposed local plan of failure

A post-processor integrated in the FLAC code makes it possible to find the critical failure surface automatically and to display the corresponding _ safety factor.

3.2 *Surface of failure locally optimized*

Let us consider a unit element of slope with properties c, (|> subjected to a given system of normal and shear stresses (Fig. 2). Assume that the failure surface is a straight line at slope angle 6. Based on the definition of the *factor of safety,* we can write:

FS = Resisting Shear Force I Driving Shear Force

$$FS = \frac{c.1.\cos\theta + \sigma_f.1.\cos\theta.\tan\varphi}{\tau_f.1.\cos\theta} \quad (3)$$

$$FS = \frac{c+\sigma_f.\tan\varphi}{\tau_f} \quad (4)$$

with:
- c = cohesion;
- ϕ = friction,
- θ = angle of failure;
- σ_1, σ_3 = principal stresses;
- FS = local safety factor.

The values ofot, τ_f are related to principal stresses σ_1,σ_3; substituting into the Equation 4, we get:

$$FS = \frac{c + (\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}.\cos 2\theta) * \tan\varphi}{\frac{\sigma_1-\sigma_3}{2}\sin 2\theta} \quad (5)$$

where Of, Tf are normal and shear stresses on the failure surface (Fig. 2).

In order to obtain the minimum value of FS, 9 must yield Equation 6:

$$\frac{dFS}{d\theta} = 0 \qquad (6)$$

Equation 6 İs complicated and we used MATHEMATICA (Wolfram, 1999) (Fig. 3) to find the exact solution of 9 (surface of failure). The following expression was obtained:

$$\theta = 0.5 \ \operatorname{arccot} \left[\frac{(\sigma_1 - \sigma_3)\sec\varphi\,(2c\cos\varphi + (\sigma_1 + \sigma_3)\sin\varphi)\tan\varphi}{2\sqrt{(c + \sigma_1 \tan\varphi)(c + \sigma_3 \tan\varphi)}\sqrt{(2c + (\sigma_1 + \sigma_3)\tan\varphi)^2}} \right] \qquad (7)$$

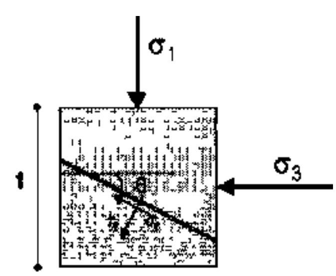

Figure *2.* Stresses acting on a unit element.

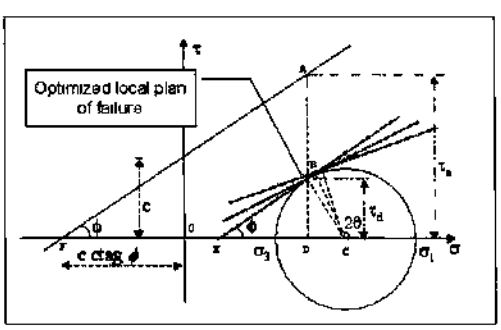

Figure 3 Stress state on Mohr's circle-optimized local plan of failure

If G from Equation 7 is substitued into Equation 5, we obtain the minimum factor of safety for a particular element corresponding to a stress tensor or point with a given stress state.

A simple program with MATHEMATICA was used to compare the results of the two approaches.

4 FLAC CODE PRESENTATION

The FLAC code, developed by ITASCA (1996), is an explicit finite difference code which simulates rock or soil structures which undergo plastic flow when their yield limit is reached. Materials are represented by two-dimensional grid elements which, in response to applied forces or constraints, follow a linear or non-linear stress/strain law. If stresses are high enough to cause the material to yield and flow, the grid elements deform and move with the material it represents. The Lagrangian calculation scheme used is well suited to modeling large distortions. In addition, the time-stepping approach to the solution of the equations of motion at each element node allows the user to examine the development of yield (or material collapse) as it develops instead of visualising the end (equilibrium) state only.

5 PROGRAM INTEGRATED IN FLAC

This program is developed with the FISH programming language (associated with FLAC), based on the "imposed failure surface" approach for slope stability analysis through a circular failure. It can be used for the determination of the safety factor using the stress state computed from the finite difference method.

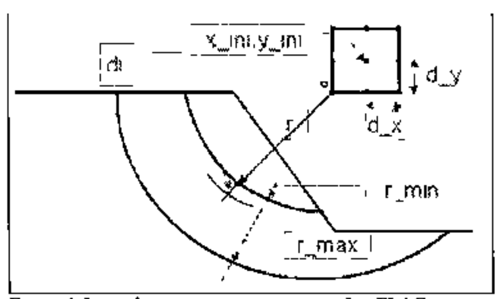

Figure 4. Input data into program integrated in FLAC

Some features of this program are:

the input data of the stress state İs determined by FLAC calculation,

it realizes the calculation of the minimum factor of safety through a procedure of automatic calculation according to a user's defined grid (Fig-4),

a graphic plot showing the failure surface is provided.

6 APPLICATION TO A REAL CASE

6.1 *Description of the site*

The example corresponds to a slope of an open pit mine in France. The site is the Antoinette pit located south of the square of Mercoirol under the old Departmental Path 906. The exploitation of this pit started in May 1992 and stopped in 1993. Its surface is 26 hectares with a maximum depth of 90m. It has a rectangular form, being approximately 400m İn length and 300m in width. After the end of mining, the water level was established at the bottom of the pit with the same dimensions (Hadadou & Alheib, 1999).

Figure 5 Antoinette Pit

6.2 *FLAC Model and stability analysis*

We obtained a representative open pit cut, in a direction perpendicular to the slope. It is representative of the pit. This cut is considered geometrically the most unfavorable. The average slope angle İs close to 50°. The slope is composed of hard stratified rock beds.

A 110x66 grid was used as shown in Figures 6 and 7. No horizontal displacement was allowed on the vertical boundaries, while the bottom boundary was completely fixed in both vertical and horizontal directions. A Mohr-Coulomb constitutive model was assigned as described to all zones with the following properties (Table 1):

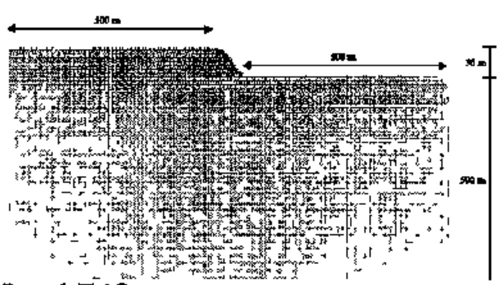

Figure 6 FLAC zone geometry

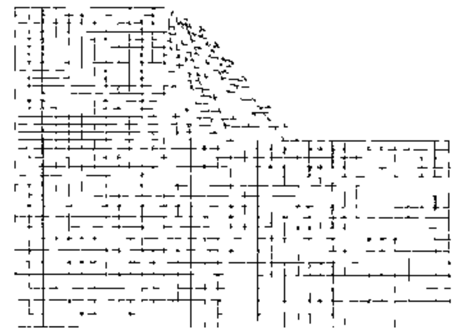

Figure 7. Finite difference mesh for the slope analyzed

Table 1 Material properties

Properly	*Value*
density (kN/m3)	27
friction angle (°)	35
cohesion (kPa)	100

Our objective was to study the long-term behavior of the slope.

6.3 *Results and discussion*

Below are the results of the calculation of the local and global safety factors; the failure mechanism for the pit is illustrated in Figure 8, which also shows the circle of failure corresponding to a factor of safety equal to 1.33 (Tables 2 and 3).

The contour lines of the local factors of safety are illustrated in Figure 9, which shows that a localized area of weak zones is developed within the model.

This same example was solved with the limit equilibrium method in order to compare the results obtained with the solution of Bishop (Hunger, 1988).

Table 2 Results of FLAC code

FS - global	1 33
x-coordinate of the center of the circle	574 m
y-coordinate of the center of the circle	570 m
diameter of the circle	75 m

Table 3. Minimal and maximal values of the local factor of safety

FS - local minimum	1.005
FS - local maximum	5.490

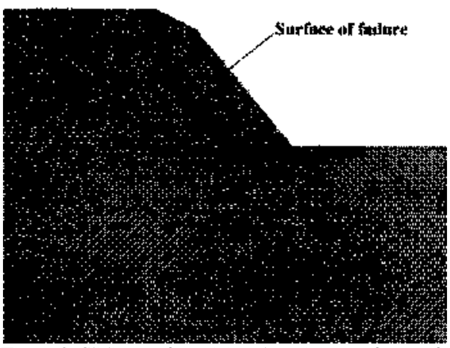

Figure 8. Sliding surface giving the minimum factor of safety.

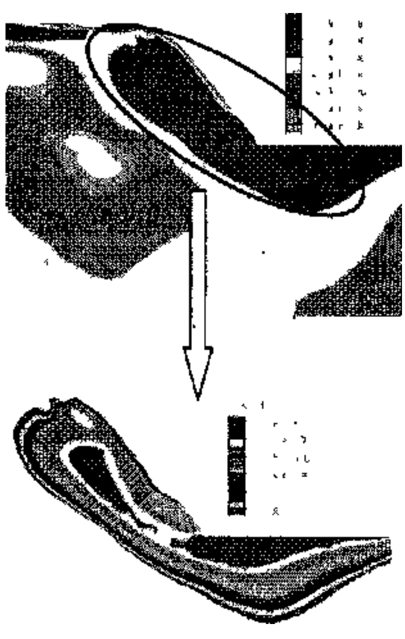

Figure9. Isocontours of the local factor of safety (ex_3 _local FS)

Figure 10 below illustrates the circle of failure corresponding to the minimum factor of safety, which is equal to 1.5.

In both cases, the analysis of the stability of the pit in the long term shows that the slope does not present a major problem of instability. The safety factors obtained are largely higher than 1. On the other hand, the difference in global FS (=11%) values may be explained by the nature of the two approaches: the Bishop simplified method tends to be conservative because it neglects internal strength.

It is more reliable to use a numerical method than sliced segments, and accurate stress results were achieved. The boundary condition assumptions and the choice of element size and shape are significant parameters in the simulation.

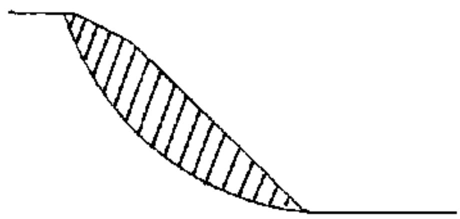

Figure 10 Circle of failure by Bishop method (CLARA program)

7 CONCLUSIONS

Limit equilibrium is the basis for most of the stability analysis procedures used in geotechnical engineering. For more complicated problems, it is appropriate to perform limit equilibrium analysis using a numerical method. The development has a number of advantages over slope stability analysis:

- more complex phenomena can be modeled (effect of water, dynamic, discontinuity, etc.);
- capacity to introduce constitutive models;
- factor of safety is not constant along the slip surface;
- the factor of safety of a slope can be computed with FLAC by reducing the soil shear strength in stages until the slope fails, but this does not show a well-defined surface of failure. The program developed can display two-dimensional plots of die sliding plans, İsozones of local factors of safety, in addition to calculating the global factor of safety.

In the future, progress is expected m improving the method of examining failure surfaces without assuming a failure mode in advance.

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