17th international Mining Congress and Exhibition of Turkey- IMCET 2001, ©2001, ISBN 975-395-417-4

Failure and Failure Theories for Anisotropic Rocks

E. Yaşar
Department of Mining Engineering, Çukurova University, Adana, Turkey

ABSTRACT: This paper deals with the failure mechanism of sedimentary rocks in terms of underground stability. The engineering behaviour of anisotropic rocks and anisotropic influence needs further investigation with regard to stability analysis due to the different failure mechanisms. In order to understand the effects of anisotropy, laminated rocks were investigated. Laboratory test results for the anisotropic strength characteristics of sedimentary rocks in different directions were determined. The material selected for the tests was sandstone and siltstone from the Upper Miocene-Pliocene Handere Formation (Th) in n«>ithem Adana (Turkey). Samples were then prepared as test samples to ISRM standards with the samples ot stated at 10° increments to the lamination plane for the application of failure theories. The deft[1] mation characteristics of these anisotropic rocks in the application of the failure theories were determined by the lamination plane angle to the loading direction when applying the stress state. Various failure théorie», were applied to determine the anisotropic strength and deformation properties of these rocks. It is commonly assumed that bedding planes of sedimentary rocks exhibit homogeneous properties and it was found to be a transversely isotropic rock material.

1 INTRODUCTION

A great number of underground constructions are built in laminated sedimentary rock formations. Changes in the engineering properties of rock masses depend on the mineral content, mineral orientation and grain size, density, discontinuity, lamination and bedding planes. Experimental works show that the mechanism of failure varies according to the conditions of stress and loading angle to lamination in the specimen. Because of the different degrees of strength in respect of the loading direction to lamination, laminated siltstone and sandstone specimens were prepared with 10 different angles at 10° increments between 0° and 90°. Several uniaxial compressive, indirect tensile strength and triaxial tests were conducted for each angle. The engineering properties and failure mechanisms were determined with respect to the different directions of 0° to 90 in the laminated rock specimens. A complete picture of the laminated rock behaviour may be predicted from the different loading angles. The failure plane developed along the lamination and other discontinuities.

The laboratory test results for the anisotropic strength characteristics of sedimentary rocks in different directions were determined. A variety of failure criteria for anisotropic materials have been proposed. The applied theories have been classified into three groups: mathematical continuous criteria, empirical continuous models and discontinuous weakness plane theories. The deformation characteristics of these anisotropic rocks for application to the failure theories were determined by the lamination plane angle to the loading direction when applying the stress state. In order to obtain the friction (φ) and cohesion (Co) parameters required for each (ß) orientation, the empirical Hoek-Brown failure criterion was fitted to the triaxial data sets for each orientation. The corresponding Mohr-Coulomb failure envelope and friction and cohesion parameters were derived using Balmer's equations as per the procedure detailed by Hoek et al. (1980). The single plane of weakness theory proposed by Jaeger is the most widely known. In this theory, the classic Mohr-Coulomb criterion is used to describe the failure of both the bedding planes. Good results can be obtained for stratified rocks with this criterion.

2 METHODOLOGY OF MEASUREMENT

The material selected for the tests was sandstone and siltstone from the Upper Miocene-Pliocene Handere Formation (Th) in northern Adana (Turkey). It is characterised by lamination planes. The Handere Formation, which is whitish, yellowish, gray and black İn colour, consists mainly of four units: siltstone, sandstone, fossilized limestone and gypsum. The Handere Formation has concordance and transitive contact with the Kuzgun Formation, which is below it. Quaternary units have discordance and are above tie Handere Formation. Because of the covering of the Quaternary units, the thickness of this formation was not measured. The clastic grains consist predominantly of quartz (42%), rock fragments (22%) and feldspar (10%). The intergranular substance consists of matrix (18%), cement (4%), mica fragments (2%) and traces of coal fragments, altered feldspars and rock fragments. The size of the grains varies from 10 to 20 Jim. There were no signs of weathering on the rock specimens. The sandstone was generally weaker than the siltstone. The samples were taken from the more uniformly laminated sections.

The triaxial tests were applied to measure the variation in failure strength in different loading angle orientations and confining pressures for the analysis of structural anisotropy. A large laboratory testing programme was designed for this rock type in order to investigate the material strength anisotropy. A total of 400 tnaxial compression tests were performed for the loading orientations β = 0, 10, 20, 30, 40, 50, 60, 70, 80 and 90° and with the confining pressures σ_3= 0, 5, 10, and 15 MPa. The definition of the angle ß is given in Figure 1. Direct observation of the sample failure surfaces revealed that the failure mode had a strong dependence on loading orientation and confining pressure. Preliminary tests were performed to determine the degree of structural anisotropy of the material. First, wave velocities were measured along three identified structural axes of the rock (Fig. 1). According to the results, the lowest wave velocity occurred in the Z direction normal to the lamination, while the highest occurred İn one direction parallel to the lamination. Moreover, the difference between the velocities measured along two orthogonal directions (X, Y) in the lamination was very small. After calculating the anisotropy degrees from velocity data obtained from three cubic samples, it was concluded that the principal anisotropy of the material is related to the presence of bedding planes and the secondary anisotropy within bedding planes can be neglected. Therefore, the material behaviour can reasonably be considered transversely isotropic in nature. This can be summarised as follows.

Failure depends on confining pressure for the orientation ß = 0° and 90°. The sample failure takes place by bursting of the lamination under low confining pressures.

For orientations between $\beta = 20°$ and $\beta = 70°$, the failure is clearly controlled by sliding of the lamination.

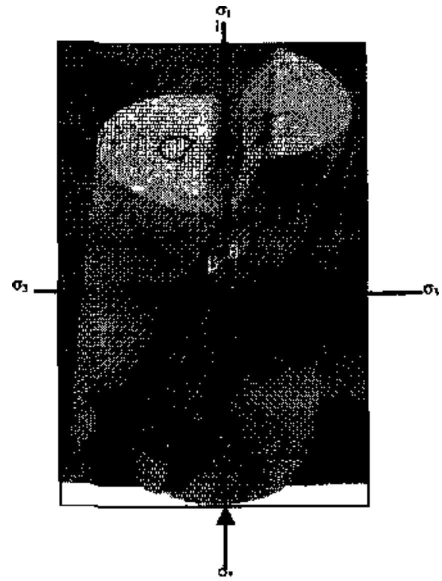

Figure 1. Loading direction on the samples.

The values of the failure strength defined at the peak point of stress-strain curves are presented both with loading orientation and confining pressure. There was a large variation in strength with the loading orientation for all the confining pressures tested. The material failure is due to sliding along the lamination for loading orientations between |$ = 20° and ß = 70°. The sudden decrease in strength at these angles in the rock samples represents the transition from parallel lamination failure to perpendicular lamination failure. The degree of material strength anisotropy is commonly quantified by the ratio of the largest and smallest values of failure stress for a given confining pressure.

$$\sigma_{an} = \sigma_c \max / \sigma_c \min \qquad 0)$$

The experimental values of this anisotropy ratio for various confining pressures are presented in Figure 2. The value of σ_{an} continuously decreases with confining pressure. This means that the material anisotropy İs smaller when the confining pressure is higher. However, the strength anisotropy ratio of the laminated rocks seems to show a tendency towards a constant value of about 1.1 even under high confining pressure.

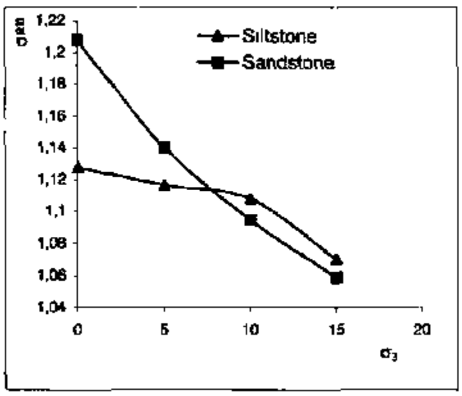

Figure 2. The ratio of anisotropy in the test material.

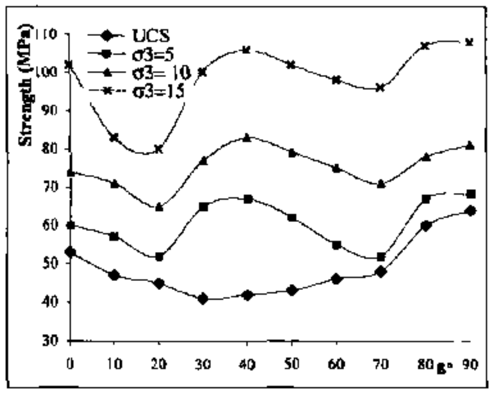

Figure 3. Test results for sandstone

3 TEST RESULTS

Sandstone and siltstone from the Hanaere Formation were studied in order to determine the geological, physical and mechanical properties of anisotropic rocks using a testing macnine. The samples were then prepared as test samples to ISRM standards with the samples orientated at 10° increments to the lammation plane for application to the failure theories. A series of triaxial and uniaxial tests were then conducted on each sample inclination, with repetition of the tests where matenal was available. A total of 400 sandstone and siltstone samples from the Handere Formation were tested. This study was concerned with the application of theories of anisotropic strength (Figures 3, 4 and 5). Tables 1 and 2 present a summary of the tests for sandstone and siltstone respectively.

Indirect tensile tests were also conducted using Brazilian discs. Here, the 90° core was prepared as a batch of discs which were then tested with the bedding at different inclinations. It was assumed that the induced tensile breaking stress generated m the test was at 90° to the applied load and, therefore, the tensile strength measured was also in this orientation.

Table 1 The results for sandstone

β	UCS (MPa)	σ_3=5 (MPa)	σ_3=10 (MPa)	σ_3=15 (MPa)	φ°	τ_0
0	53	60	74	102	37	13
10	47	57	71	83	36	12
20	45	52	65	80	32	II
30	41	65	77	1002	35	10.5
40	42	67	83	106	34	10
50	43	62	79	302	36	10
60	46	55	75	98	35	108
70	48	52	71	96	31	10 13
80	60	67	78	107	36	13
90	64	68.4	81	108	42	135

Table 2 The results for siltstone.

β	UCS (MPa)	σ_3=5 (MPa)	σ_3=10 (MPa)	σ_3=15 (MPa)	φ°	τ_0
0	76	86	102	129	45	14
10	69	79	87	100	43	13 5
20	67	74	80	92	39	12 52
30	67	87	101	115	41	13 48
40	72	88	105	112	42	125
50	74	93	112	124	41	12
60	76	87	103	120	40	14
70	68	79	97	116	41	145
80	87	101	120	131	46	15
90	85 71	96	113	138	49	14

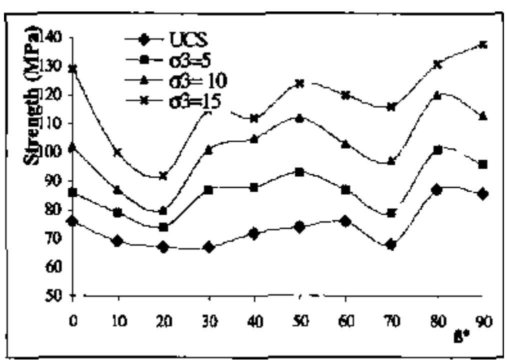

Figure 4 Test results for siltstone

4 APPLICATION OF THE TEST DATA TO THE THEORIES

Four failure theones were used for interpretation of the laboratory test results for the anisotropic rocks: the single plane of weakness theory (Jaeger, 1960); the continuously variable shear strength theory (Jaeger, 1960); the Walsh-Brace theoiy (1964); and the variable friction angle and cohesive strength theory (Donath, 1972). The uniaxial test results for the anisotropic strength characteristics of the Handere Formation were determined and applied to the failure theories.

The single plane weakness failure theory for uniaxial conditions was determined from the following expressions (Jaegar and Cook, 1979)"

$$\sigma_1 = \frac{2\tau_0}{(1-\tan\phi\cot\beta)\sin 2\beta} \qquad (2)$$

The failure stress di can be calculated by specifying τ_0' and $\tan\phi'$(constant for varying orientations of ß). It is necessary to evaluate two cohesive strength parameters and two coefficients of internal friction (τ_0', τ_0, $\tan\phi'$ and $\tan\phi$) for anisotropic materials.

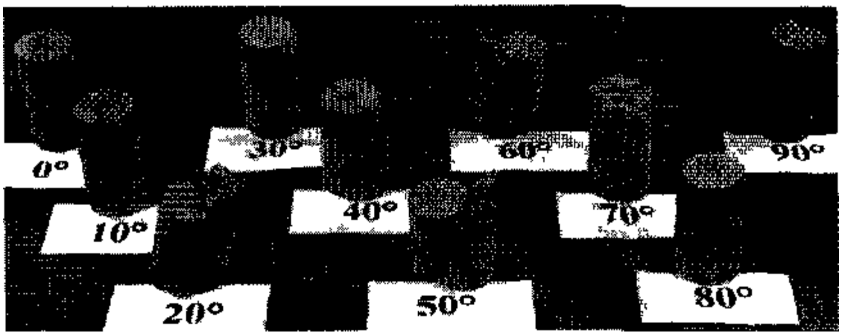

Figure 5. Samples after tnaxial testing

Walsh and Brace (1964) proposed a failure criterion which modified Griffith's tensile failure theory. The failure stress (0VÖ3) for a randomly oriented long crack is:

$$(\sigma_1-\sigma_3)_L = \frac{C_0[(1+\tan\phi_L^2)^{1/2}-\tan\phi_L]+2\tan\phi_L\sigma_3}{2\sin\alpha\cos\alpha(1-\tan\phi_L\tan\alpha)} \qquad (3)$$

For calculating the weakest fracture strength, a friction angle of 45° was used.

The failure criterion for the continuously variable cohesive strength theory is as follows:

$$(\sigma_1-\sigma_3) = \frac{2\tau_0 - 2\sigma_3\tan\phi}{\sec\phi-\tan\phi} \qquad (4)$$

where $\tau_0 = A - B[\cos 2(\beta'-\beta)]$

A and B are constants,

$\beta' = 45°$,

ϕ = angle of internal friction.

The variable friction angle and cohesive strength theory assumes that material fails in shear and has a variable cohesive strength (τ_0) and a variable internal friction angle, $\tan\phi$ (Donath, 1972).

The value of the internal friction angle ($\tan\phi$) may vary with respect to ß. The variation of the internal friction angle can be calculated by the following equation:

$$\tan\phi = C - D[\cos 2(\beta'-\beta)] \qquad (5)$$

C and D are constants,

$\beta' = 45°$.

Figure 6 illustrates the application of the four failure criteria to test data for sandstone. Figure 7 illustrates the application of the four failure criteria to test data for siltstone. Tables 3 to 6 give the constants derived for each failure theory. The β orientation when the strength was at a minimum was found to be 70° for both sandstone and siltstone. The theoretical Walsh and Brace and single plane of weakness failure criteria, when applied to die test data, generated similar strength envelopes with both predicting a 'U'-shaped reduction in strength for failure along the weakness plane. At most confining pressures, these two criteria overestimated the strength in the regions where they predicted failure through the intact rock. However, for sandstone tested at confining pressures of 10 MPa and 15 MPa, these two theories produced a reasonable fit to the test data. This seems to indicate that at higher confining stresses there is a change in the failure mechanism of the sandstone and siltstone. The continuously variable cohesive strength criterion and the variable friction angle and cohesive strength criterion produced failure envelopes that predicted a continuous change İn strength in relation to the ß orientation. The variable cohesion theory tended to either underestimate or overestimate the strength at ß = 90° and β = 0°. The variable friction and cohesion theory provided a better fit to the data. It was also observed that generally σ_1 for β = 0° is lower than CTi for ß = 90°, again except for sandstone tested at confining pressures of 10 and 15 MPa, Figures 3 and 4. In these cases, both of the empirical failure criteria underestimated the actual strength.

Table 3. Single plane of weakness parameters

	Weak Plane β = 70°		β = 0°		β = 90°	
	c (MPa)	φ°	c (MPa)	φ°	c (MPa)	φ°
sandstone	11.25	37	12	37	13.59	37
siltstone	13.78	45	15	45	17	41

Table 4. Walsh and Brace parameters.

	Weak Plane β = 70°		β = 0°		β = 90°	
	UCS (MPa)	φ°	UCS (MPa)	φ°	UCS (MPa)	φ°
sandstone	42.75	37	53	37	64	37
siltstone	68.81	45	76	45	85.71	45

Table 5. Continuously varying cohesion parameters.

	φ°	'A' constant	'B' constant
sandstone	37	15	3.5
siltstone	45	17	5

Table 6 Continuously varying friction and cohesion parameters.

	A	B	C	D
sandstone	15	3.5	42	6
siltstone	17	5	49	7

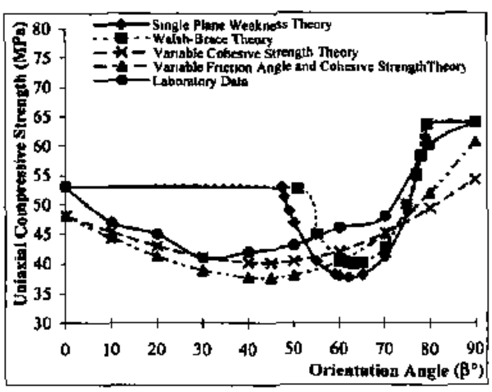

Figure 6 The application of failure theones to sandstone rock specimens in different bedding planes.

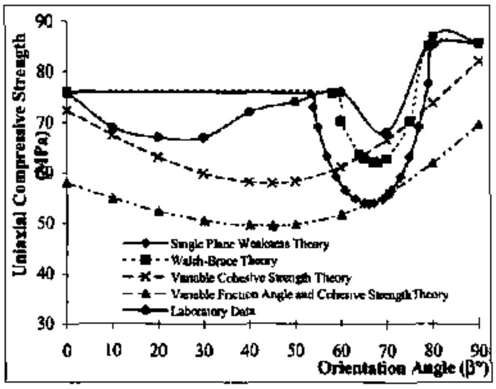

Figure 7. The application of failure theones to siltstone rock specimens m different bedding planes.

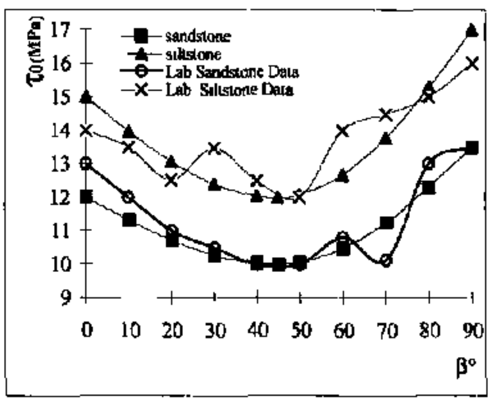

Figure 8. Cohesion strength versus bedding planes.

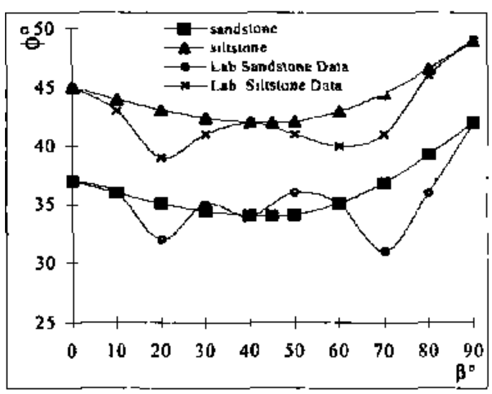

Figure 9. Friction angle versus bedding planes

When this theory was applied to both sets of sandstone and siltstone data, it was observed from Figures 6, 7, 8 and 9 that the method predicted the occurrence of the minimum fracture strength at a much steeper angle of anisotropy than the observed behaviour of the test data. This was also true of the Walsh-Brace failure theory, in which a similar description of failure was evident by the single plane of weakness theory.

However, when a spatially varying cohesion was introduced with a constant friction angle, the results indicated a closer theoretical evaluation of the measured minimum value of anisotropy at ß = 45° for each of the sandstone and siltstone data sets. The model, however, significantly underestimated the strength of both data sets at orientations of 0° and 90°.

The method that incorporated a variable friction angle as well as a variable cohesion in equation 4 produced the best overall fit to the sandstone and siltstone laboratory data. The model results predicted a minimum strength close to the test data at ß = 45° as well as producing a much improved estimation of the fracture strength at orientations of 0° and 90°.

4.1 *Effect of anisotropy on (ensile strength*

Figure 10 illustrates the change in tensile strength with respect to ß orientation for sandstone and siltstone. The figure illustrates that there is a decline in tensile strength from ß = 90° to ß = 0°. The scatter in the data precludes a close correlation of a regression curve to the data points; therefore, a linear trend line was fitted. The ratios between the tensile strength at ß = 90° and ß = 0° are 0.86 and 0.84 for sandstone and siltstone respectively.

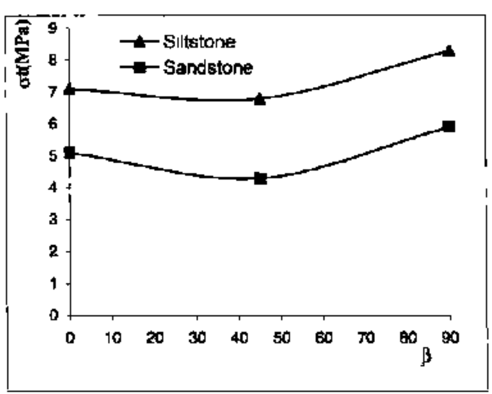

Figure !0. Change in tensile strength with respect to ß.

For each confining pressure, multiple tests were undertaken for any particular ß orientation. Multiple tests were also carried out at each ß orientation for the tensile and UCS tests. The data obtained from these tests exhibited variability in the strength of the rock tested at the same ß orientation and under the same conditions. This variability is a natural phenomenon of rock material and is attributed to the natural variation of rock strength and variation in lamination properties such as thickness, spacing, strength, orientation and form within each siltstone unit.

5 ANISOTROPIC STIFFNESS BEHAVIOUR

5.1 *Elastic properties and anisotropy*

The sandstone and siltstone of the Handere Formation are usually stratified deposits formed from sedimentation consisting of a cyclotherm sequence of various rocks with different material properties. The deformatipna! properties of diese rocks depend upon the orientation of die bedding planes and laminations to the major principal stresses. Since the siltstone and sandstone of die Handere Formation are located in stratified layers, it is reasonable to assume from the test data that such materials exhibit anisotropic elastic deformation behaviour that can be best approximated as a transversely isotropic material.

The transversely isotropic analysis represents a stratified elastic medium in which there are distinctly different elastic moduli perpendicular and parallel to tie layers. Five unique moduli are required to specify transversely isotropic material: E_{xz}, E_y, v_{xz}, v_y, G_{xy}. The shear modulus between the plane of isotropy and normal plane G_{xy} is very difficult to determine experimentally using direct testing metiiods (Chen et al., 1993). Laboratory data for sandstone and siltstone can be utilised for E^ and E_y with v_{X7}. and v_y assumed to be 0.2 to empirically determine $G_{„v}$ iteratively using a simple modelling technique. An example of the shear modulus calculated using the modelling method for sandstone is given in Table 7.

5.2 *Effect ofanisotropy on Young's modulus*

The laboratory test results show the change in Young's modulus with respect to ß orientation for sandstone and siltstone. A linear regression line to each set of data indicates a general decline in Young's modulus from ß = 90° to ß - 0°. Further testing is required to produce a better relationship between Young's modulus and ß orientation-

Table 7. Example of use of laboratory test data.

β	UCS(MPa)	UTS(MPa)	E(GPa)	K(MPa)	τ_0(MPa)	ϕ	Gxy(GPa)
Sandstone							
0°	53	5.1	16.8	169	13	37	
45°	42	4.3	14.2	131	10	34	5.047
90°	64	5.9	17.4	202.5	13.5	42	
Siltstone							
0°	76	7.1	17.8	190.5	14	45	
45°	72	6.8	15.1	149.7	12	42	3.628
90°	85.71	8.3	18.3	227.8	14	49	

Young's modulus for rocks can be determined from failure stresses obtained in triaxial tests with ß = 90° and ß = 0°, as in these orientations, and the failure takes place in the rock matrix. However, according to experimental data for many anisotropic rocks, the strength with ß = 90° may be clearly different from that with ß = 0°. Therefore, it is useful to take different values of the cohesion and friction for the two principal directions. Experimental determination of these rocks' cohesion strength and friction angle in two principal orientations was carried out. However, the cohesion and friction of weakness planes have to be determined from failure stresses determined in triaxial tests with the loading orientation resulting in the minimum strength, generally for 20°< ßnn< 70°. At such an orientation, the material failure occurs along weakness planes.

6 CONCLUSIONS

The uniaxial compressive strength behaviour of siltstone and sandstone from the Handere Formation is dependent upon the orientation of the plane of anisotropy to the applied stress.

The maximum compressive strength occurred at an orientation of ß = 90° and 9 = 0° with a minimum value of strength occurring at ß = 68° and 8 = 22° for Jaegar's single plane of weakness theory and the Walsh-Brace criterion respectively.

The variable cohesive strength TO and the coefficient of internal friction tan0 vary with the orientation of anisotropy and they produced the best fit to the data obtained in the laboratory.

It was clear from the laboratory data that the modulus of elasticity varied with the orientation of the plane of anisotropy. The shear modulus between the plane of isotropy and the normal plane for both the siltstone and sandstone data sets had a reduction factor of 0.46 and 0.53 from the modulus of elasticity measured İn the laboratory for ß = 45° using transverse isotropy.

Triaxial testing and measurement of all directional deformation modulus properties are proposed and the anisotropic strength and elastic theories should be applied to more comprehensive and detailed data sets for all major Handere Formation lithology.

The variable friction and variable cohesion theory produced the best fit to the triaxial data, with the exception of sandstone and siltstone tested at confining pressures of 10 and 15 MPa. For these two cases, the single plane of weakness and Walsh-Brace criterion model best fitted the test data.

The maximum compressive strength for both the sandstone and siltstone occurred at an orientation of ß = 90° with a minimum value of strength occurring at ß = 70°. Jaegar's single plane of weakness criterion and the Walsh-Brace criterion do not have the flexibility and advantage of the variable friction angle and cohesive strength criterion, which is considered the most appropriate in most cases for modelling laboratory results. However, the test results for the sandstone and siltstone tested at confining pressures of 10 MPa and 15 MPa were best fitted by the single plane of weakness and Walsh-Brace theories. The sandstone was the weaker of the two lamination rocks tested and the test results may be a reflection of changing failure mechanisms at higher confinement. In order to validate this hypothesis, further triaxial testing at higher confining stresses is required.

It was observed from the laboratory data that the modulus of elasticity varied with the orientation of the plane of anisotropy. There was a large scatter in the test data and further testing is required to determine more accurately the influence of anisotropy.

This paper İs an introduction to the preliminary phase of a more detailed programme of research involving the detailed testing of a wide range of sandstone and siltstone samples.

REFERENCES

Chen, D.H., Zaman M.M. and Kukreti, A.R. (1993). Laboratory Testing and Constitutive Modelling of Coal Including Anisotropy, Structure and Properties of Engineering Materials. Vol. 48 pp 349-354.

Donath, F.A. (1972). Strength Variation and Deformational Behaviour in Anisotropic Rocks, State of Stress in the

Earth's Crust, American Elsevier Publ Comp, New York pp 281-298.
Hoek E. And Brown E T., (1980) Empirical Strength criterion for rock mass J. Geotech. Div. Am. Soc. Civ. Eng. 106. 1013-1035.
Jaeger, J.C. and Cook, N G W (1979) Fundamentals of Rock Mechanics, Chapman and Hall, London, Third Edition
Jeager, J C, (1960). Shear Failure of Anisotropic Rocks. Geological Magazine, Vol 97, pp65-72.
Walsh, J B. and Brace, W F , (1964). A Fracture Criterion for Brittle Anisotropic Rocks. Joum. Geophysical Research, Vol. 69(16), p3449