

**NUMERICAL MODELLING OF SURFACE SUBSIDENCE  
ARISING FROM LONGWALL MINING OF STEEPLY INCLINED  
COAL SEAMS.**

**EĞİMLİ DAMARLARDA UZUNAYAK MADENCİLİĞİNDEN  
KAYNAKLANAN YERYÜZÜ TASMANININ İN SAYISAL  
MODELLEMESİ**

**AFSARI NEJAD, M. and REDDISH, D. J.** *School of Chemical, Environmental and  
Mining Engineering University of Nottingham, NG7 2RD, UK*

**ABSTRACT**

This paper presents results from and the methodology of a numerical modelling investigation into the surface ground movements above longwall mining of inclined and steep seams with varying panel configurations. A modelling approach was developed using a finite difference numerical model Fast Lagrangian Analysis of Continua (FLAC). On the basis of this methodology, representative surface subsidence profiles were simulated and the results of simulations were validated against the UK data using the Subsidence Engineer's Handbook (SEH) and influence function methods (Ren et al, 1989). Furthermore, the proposed methodology was applied to two UK case histories for validation purposes.

**ÖZET**

Bu çalışmada, dik ve eğimli damarlarda hazırlanan farklı pano büyüklüklerindeki uzunayakların çalışması sonucu oluşabilecek yeryüzü hareketlerinin belirlenmesine ilişkin gerçekleştirilen sayısal çözümleme yöntemlerinin sonuçlarına yer verilmektedir. Sayısal modellerin çözümünde, Sonlu farklar metodunu kullanan FLAC programından yararlanılmıştır. Çalışmadan elde edilen tasman profilleri, benzer problemler için İngiltere koşulları gözönüne alınarak hazırlanan Tasman Mühendisinin El Kitabı'ndan elde edilen sonuçlar ve Etki Fonksiyonu Metodu'ndan (Influence Function Methods) elde edilen sonuçlarla karşılaştırılmıştır.

## 1. INTRODUCTION

Increasing world demand for energy and mineral resources has resulted in the mechanisation and introduction of rapid excavation techniques in modern mining operations. This, in turn, has given rise to severe surface and subsurface subsidence problems. Surface subsidence prediction for inclined and steep seams has received less attention than level seams due to the difficulties involved in the extraction of such coal seams. In some countries, a great deal of the coal reserves of high quality is classified as inclined and steep strata. For local economic reasons, these deposits have to be extracted, and consequently, the problem of surface subsidence of this type of deposit is still highly relevant in many parts of the world. Numerical modelling techniques, particularly the finite difference method is a powerful tool for obtaining solution to these problems. It differs from the physical modelling technique, profile function and mathematical methods in that it can take into account the mechanical properties of the overburden and analyse the failure and post failure behaviour of rock masses surrounding the mine openings in a reasonable time.

During this research, several constitutive models in FLAC were examined to accurately simulate strata movement arising from the longwall mining of inclined and steep seams (Itasca, 1995). The results of this research when validated against those predicted by the SEH prediction method (NCB, 1975) and the influence function method using the SWIFT package (Mineral Resources Engineering, 1996) showed that the asymmetric pattern of stress and displacement distributions in inclined seam situations can be accurately simulated using a combination of two anisotropic constitutive models.

## 2. MODEL SPECIFICATION

Figure 1 shows the general panel configurations and boundary conditions used in the numerical analysis. This configuration has been used effectively for surface subsidence simulation of

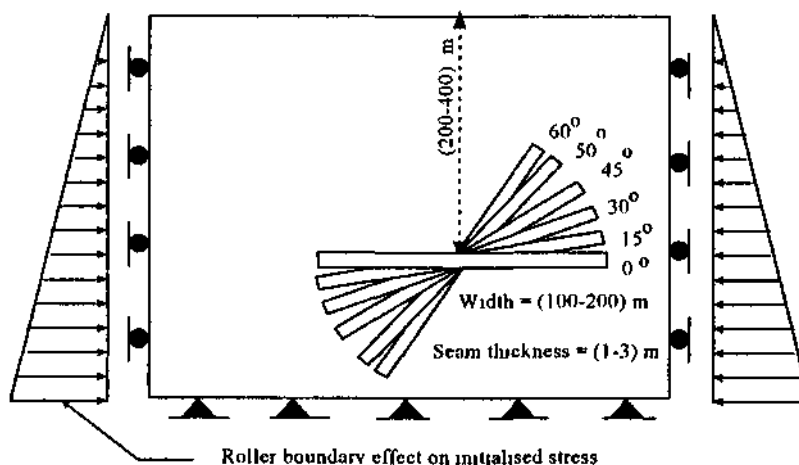


Figure 1 General panel configuration and boundary conditions

longwall panels by several investigators (e.g. Fitzpatrick, 1987; Yao et al., 1993). In situ stresses were assumed to be hydrostatic (Wilson, 1980; Trueman, 1988; Alejano et al., 1995).

### 3. Evaluation of FLAC standard models

The main initial objective of the investigation was to examine the basic capabilities of each standard FLAC model for surface subsidence simulation of inclined coal panels. To save space, the details of the material properties for each individual model are omitted at this stage. An effort was made to recognise the basic trends of strata movement suggested by different models by using the methodology recommended by established investigators (Starfield & Cundall 1988; Hoek et al. 1991). The following numerical models were evaluated:

#### Isotropic Models

- Elastic Isotropic Model
- Mohr-Coulomb Model
- Strain hardening/Softening Model.

#### Anisotropic Models

- Transversely Isotropic Elastic Model
- Ubiquitous Joints Model

Several longwall panels with different configurations were simulated using these constitutive relations. The evaluation of these initial results led to the following important conclusions:

- The position of maximum surface subsidence regardless of isotropic material properties was always above the centre of panel rather than on the down dip side which is the case for inclined seams (Whittaker and Reddish, 1989).
- For gently inclined seams, the transversely isotropic model can realistically simulate the general trend of strata movement; however, it underestimates the magnitudes of surface subsidence and horizontal displacements because of the lack of a failure criterion in the model.
- A ubiquitous joint model is better able to partially simulate the realistic trend of strata movement provided that the extent of the yielded zone is widely spread over the grid. In other situations, however, its behaviour is similar to an isotropic model.

On the basis of this analysis, it was concluded that none of the FLAC standard models could accurately simulate the asymmetric pattern of strata movement in inclined seam situations. It was concluded a combination of a transversely isotropic and ubiquitous joints models would provide a realistic approach for simulation of stable and failed

areas in inclined seam situations. This technique has been successfully applied by Alejano et al (1995) for surface subsidence analysis of case histories in Spain. Here, an effort was made to develop a systematic methodology to simulate surface subsidence above a wide range of inclined and steep longwall coal panels.

#### 4. COMBINATION OF A TRANSVERSELY ISOTROPIC MODEL WITH A UBIQUITOUS JOINTS MODEL (TU MODEL)

After an extensive investigation, it was realised that a combination of a transversely isotropic model and a ubiquitous joints model could simulate the stable and failed zones around inclined working panels, respectively. Post failure residual properties were specified by use of a specific separate function that iteratively modified yielded element parameters according to plasticity.

##### 4.1. Transversely Isotropic Model.

In FLAC implementation of this model, the state of stress is calculated using the following properties:

$E_x$  = Elastic modulus in the plane of isotropy (xoz)

$E_y$  = Elastic modulus in the direction perpendicular to the plane of isotropy

$G_{xy}$  = Shear modulus in the plane of isotropy

$\nu_{yx}$  and  $\nu_{xy}$  = Poisson's ratios

$\alpha$  = angle between the plane of isotropy and the x axis

##### 4.2. Ubiquitous Joint Model

This model accounts for the presence of a weakness plane in a FLAC Mohr-Coulomb model where yield may occur in either solid or along the weak planes, depending on the state of stress, the orientation of the weak plane as well as the material properties of both solid and weak plane. In the FLAC implementation of the ubiquitous joint model, general failure is firstly detected and relevant plastic corrections are applied as recommended for the FLAC Mohr-Coulomb model. The new stresses are used for analysis of failure on the weakness plane using the shear and tensile failure envelopes as follows:

$$f^* = \tau - \sigma \tan \phi_j + c_j \quad (\text{iff}^* > 0 \text{ shear failure in joints}) \quad [1]$$

$$f^t = s_j' - \sigma \quad (\text{iff}^t > 0 \text{ tensile failure in joints}) \quad [2]$$

where

$f$  and  $f'$  are shear and tensile envelopes,

$T$  and  $\alpha$  are shear and normal stress on the weak plane,

$\phi_j$  and  $c_j$ , and  $s_j'$  are friction angle, cohesion and tensile strength of weak plane, respectively.

## 5. DERIVATION OF INPUT PARAMETERS FOR THE TU MODEL

The following methodology for derivation of input data for numerical model proved to be a realistic approach for different panel configurations, following extensive trial and analysis

### 5.1 Transversely Isotropic Elastic Properties

With the exception of an area immediately around the excavation, the rest of the model area was treated as a transversely isotropic body. The stiffness parameters for this area were selected on the basis of applying a reduction factor (R F) to the laboratory data as presented in equation 1. This equation was developed during the course of this study for the UK Coal Measure by the authors. Poisson's ratio and shear modulus along the weak plane were estimated from Yao, et al.'s work (1993)

$$\text{Reduction Factor} = 0.00008h^2 - 0.0827h + 22.724 \quad [3]$$

where

h = mining depth (m)

### 5.2. Ubiquitous Joint Model Properties

The failed area was simulated using a ubiquitous joint model. The stiffness and strength parameters for this region were selected on the following basis

#### 5.2.1 Stiffness parameters

The Young's modulus of the rock mass was taken as an average of elastic modulus in the x and y directions which had been calculated for the transversely isotropic part of the model. Values of shear and bulk modulus were calculated using equations 2 and 3

$$G = E / [2(1 + \nu)] \quad [4]$$

$$k = E / [3(1 - 2\nu)] \quad [5]$$

where G = shear modulus (MPa)

E = elastic modulus (MPa)

K = bulk modulus (MPa)

ν = Poisson's ratio

#### 5.2.2 Strength parameters

After an extensive Numerical simulations, it was found that Hoek and Brown approach (1988), was the most realistic way for of handling strength properties in this application. According to this approach, in situ stresses increase with respect to the depth below the surface and accordingly so do the values of cohesion and the friction angle. The following

formulas were used to calculate the magnitude of cohesion and friction angle before and after failure

$$\phi = 2 \tan^{-1} \sqrt{N_{\phi}} - 90 \quad [6]$$

$$C = \frac{\sigma_c}{2\sqrt{N_{\phi}}} \quad [7]$$

Where  $\phi$  is internal friction angle (°)

C is cohesion (MPa) and

$$N_{\phi} = 1 + \frac{\sigma_c m}{\sqrt{\sigma_3 \sigma_c m + s \sigma_c^2}} \quad [8]$$

$\sigma_c$  is Uniaxial compressive strength (MPa)

$\sigma_3$  is minor principle stress (MPa) and

Constants  $m$  and  $s$  can be calculated for a disturbed rock mass using equations 9 and 10 and, for an undisturbed rock mass, equations 11 and 12

$$m_r = M \exp\left(\frac{RMR - 100}{24}\right) \quad [9]$$

$$s_r = \exp\left(\frac{RMR - 100}{6}\right) \quad [10]$$

$$m_i = M \cdot \exp\left(\frac{RMR - 100}{28}\right) \quad [11]$$

$$s_i = \exp\left(\frac{RMR - 100}{9}\right) \quad [12]$$

M is value constant  $m$  for intact rock

RMR is Rock Mass Rating

The value of RMR was back calculated from equation (13), Serafim and Pereira (1983)

$$E = 10^{\frac{RMR - 10}{40}} \quad [13]$$

E is elastic modulus in GPa ( $10^9$ N/m<sup>2</sup>)

On the basis of this technique, a program was written (called a FISH Function) and embedded within FLAC using the language facilities in the package. The additional program was executed after each five steps and renewed the values of both strength and stiffness parameters according to the state of plasticity and stresses in the ubiquitous joints model.

#### 5.2.4. Post failure stiffness properties

Bahattacharyya and Shu (1989) suggested that a higher reduction factor to in situ shear modulus than that applied to Young's modulus after failure could efficiently reflect the anisotropic behaviour of coal measures rocks. This was because of the critical reduction of shear modulus along the bedding planes. On the basis of these findings and after a systematic series of numerical model runs, a reduction factor of one-tenth (1/10) to the in situ bulk modulus together with a reduction factor one-fiftieth (1/50) to the in situ shear modulus gave the most realistic results. These reduction factors for derivation of post failure stiffness parameters from in situ properties proved to be valid for other panel configurations.

In Tables 1 to 3, the material properties calculated for a 200 m deep coal panel have been presented.

Table 1 Transversely isotropic elastic properties (200 m depth)

Ex (MPa)	(MPa)	(MPa)	Uyx	V <sub>a</sub>	P (gr/cm <sup>3</sup> )
1244	953	48	0.2	0.2	25

Table 2 Ubiquitous joint properties (200 m depth)

G(MPa) *bf *pf	K(MPa) *bf *ps	Jcoh (MPa)	Jfric (°)	Jten (MPa)	P (gr/cm <sup>3</sup> )
458 92	610 61	0.123	25	0.2	25

\* bf = before failure, pf = post failure

Table 3 Hoek and Brown parameters for calculation of cohesion and internal friction angle in ubiquitous joints model (200m depth)

m	RMR	m <sub>s</sub>	s <sub>s</sub>	m <sub>r</sub>	s <sub>r</sub>	a <sub>c</sub> (MPa)
15	14	0.695	0.00007	0.416	0.0000005	40

## 6. MODEL VALIDATION

To test the accuracy of the proposed model, the sensitivity of the model to different panel configurations were examined.

### 6.1. Sensitivity of the Model to the Width of the Panel

The sensitivity of the modelling approach to the change of panel width was examined by using the general mesh illustrated in Figure 1. The widths of panels were 100 m, 150m and 200m, respectively, while central depth, seam thicknesses and inclinations were set at 200 (m), 2 (m) and 15°, respectively. The results of surface subsidence simulation

showed an excellent agreement between the model results and those predicted by S E H and the SWIFT method. The displacement results showed a closer match with those predicted by SWIFT method than S E H. These results are presented in Figure 2.

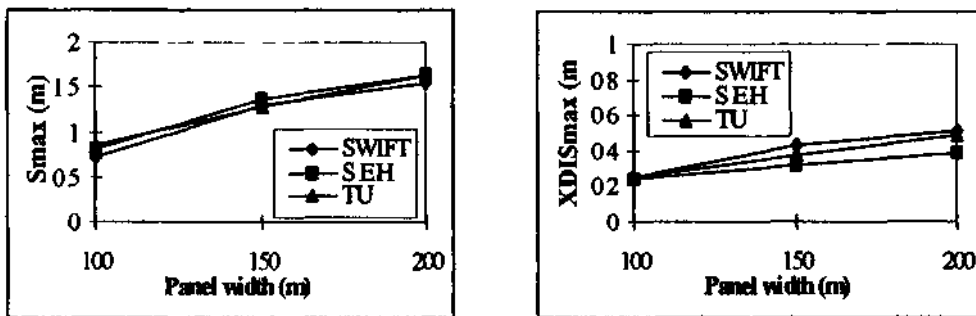


Figure 2 Comparison of maximum surface subsidence ( $S_{max}$ ) and horizontal displacements ( $XDIS_{max}$ ) for different panel widths

### 6.2. Sensitivity of the Model to the Seam Thickness

The accuracy of model to the change of seam thickness was examined using the panel configuration explained for the previous case while seam thicknesses were set at 1 m, 2 m and 3 m, respectively. These results were consistent with previous results showing an excellent agreement between subsidence troughs predicted by each of the three methods. The horizontal displacement values predicted by the numerical model again showed a good agreement with those calculated by SWIFT and both of them had greater values than those suggested by S E H. These results are presented in Figure 3.

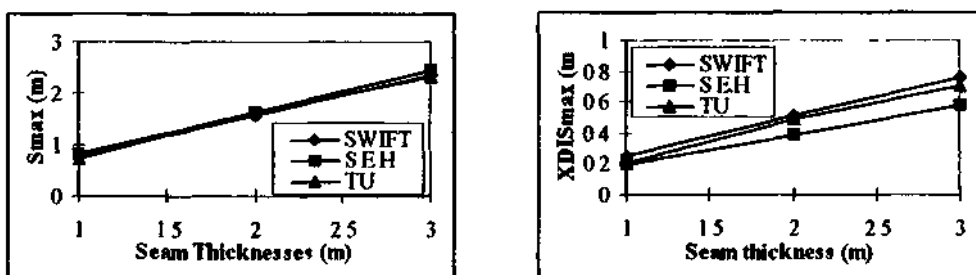


Figure 3 Comparison of maximum surface subsidence ( $S_{max}$ ) and horizontal displacements ( $XDIS_{max}$ ) for different seam thicknesses

### 6.3. Sensitivity of the Model to the Mining Depth

To test the accuracy of the model to the depth of panel, the mining depths were 200 m, 300m and 400m, respectively, while the other panel configuration were the same as previous cases. The comparison of surface subsidence results showed that the S E H had a closer match with the model results than those predicted by SWIFT. The horizontal displacement results showed that for the shallower cases the numerical model values



were greater than S E H results and closer to SWIFT values, however, there was a good agreement among all three methods. The results are presented in Figure 4.

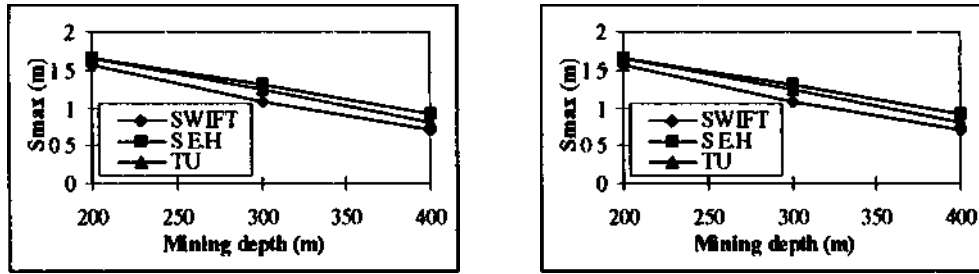


Figure 4 Comparison of maximum surface subsidence ( $S_{max}$ ) and horizontal displacements ( $XDIS_{max}$ ) for different mining depths

#### 6.4. Sensitivity of the Model to the Seam Inclination

The sensitivity of the modelling approach to seam inclination was examined using the general mesh presented in Figure 1. The results were compared with those predicted by S E H (only for cases of  $15^\circ$  and  $30^\circ$  seam inclinations) and SWIFT for steeper situations. An excellent agreement was observed for both surface subsidence and horizontal displacement results between the proposed methodology and SWIFT and S E H (only up to  $30^\circ$ ). A typical example of results for  $50^\circ$  seam inclinations has been presented in Figure 5. The comparison of the maximum surface subsidence and horizontal displacement results for different seam inclinations predicted by each method are presented (panel width 200 m) in Figures 6.

#### 6.5 Case Histories

The proposed technique was applied to two case histories from two collieries to validate the modelling results directly against measured data. A good agreement was observed between the numerical model and measured data. The results for one of these cases is presented in Figure 7.

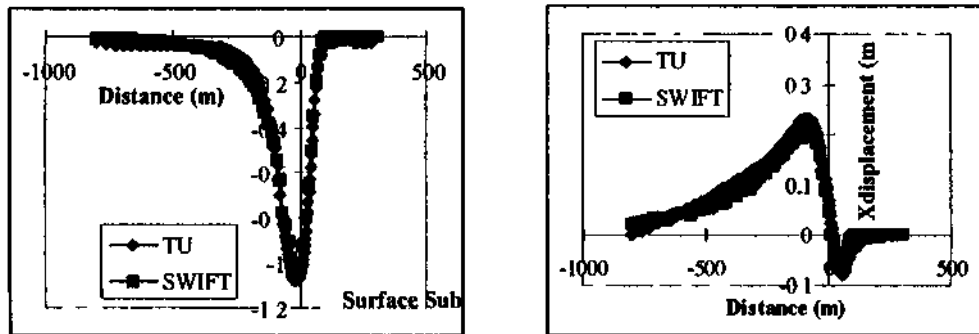


Figure 5 Surface subsidence and horizontal displacement comparisons between the Numerical model (TU) and SWIFT, central depth = 200 m, panel width = 200 m, Seam thickness = 2 m, seam inclination =  $50^\circ$

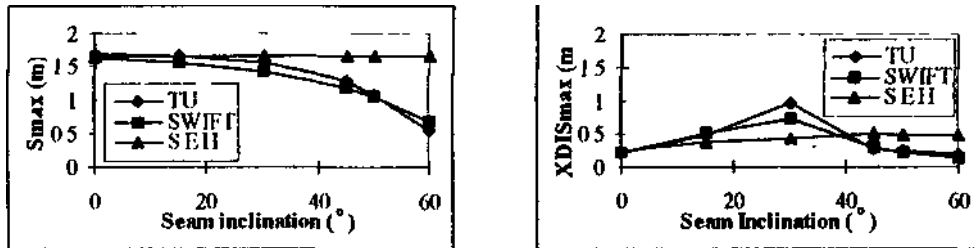


Figure 6 Comparison of maximum surface subsidence (Smax) and horizontal displacements (XDISmax) for different seam inclinations

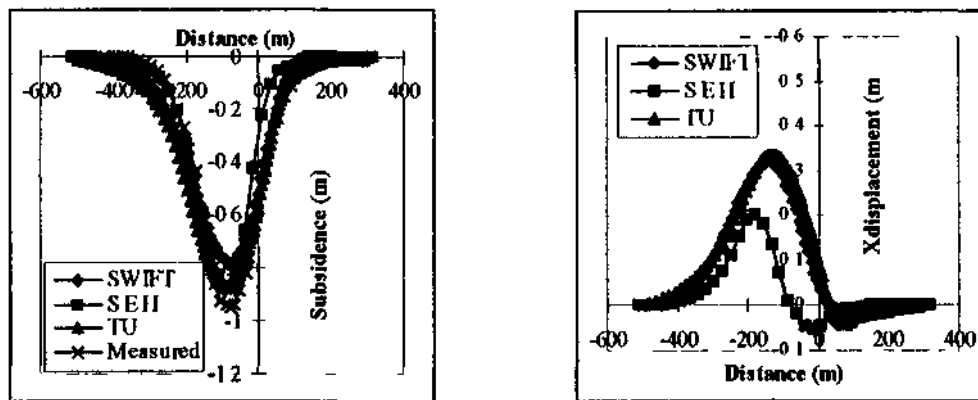


Figure 7 Case History, surface subsidence and horizontal displacement comparisons between the Numerical model (TU) and SWIFT, central depth = 254 m, panel width = 207 m, seam thickness = 1.17 m, seam inclination = 21°

## 7. CONCLUSION

A transversely elastic isotropic model can simulate the asymmetric movement above an inclined coal panel in the non caving area whereas the caving area is best simulated by use of a ubiquitous joints model

The Hoek and Brown approach can create a realistic balance between stiffness and strength parameters for the numerical simulation of inclined coal panels

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