

MULTIOBJECTIVE APPROACH FOR SOLVING THE PROBLEM OF MINIMIZING THE HORIZONTAL SURFACE DEFORMATIONS CAUSED BY UNDERGROUND EXCAVATION OF COAL SEAMS

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ABSTRACT An approach for solving the problem of minimizing the horizontal surface deformations caused by underground mining activities is presented in this paper. Two mathematical models are proposed for optimizing the dimensions and the spatial position of two working faces and a protective pillar between them. The models derived can easily be extended, and they are independent of the approaches used for predicting surface deformations. The optimization problem considered is solved by use of a multiple criteria decision method based on the Pareto-optimality. An algorithm is developed, its basic stages are described briefly, and the implementation of the computational procedure is demonstrated with a numerical example.

1 INTRODUCTION

Due to the increasing demand for coal world-wide, various surface sites, such as buildings, railway lines, canals and roads, have been subjected to undermining in recent years. Horizontal deformations of the earth surface, being one of the side effects observed as a result of these underground mining activities, often cause serious material damages, and even loss of human life. To construct buildings or installations that are adapted to the expected deformations, the following criteria are to be considered:

- 1 The horizontal deformations have to be predicted precisely before the development of the mine (this topic is beyond the scope of the present paper)

- 2 The mining method used must ensure

- 4- minimum horizontal deformations and thus minimum damages of the surface,

- 4- minimum loss of coal along with minimum damages underground

Hence, the problem of determining the coal working faces' parameters, and their position with respect to the built-up construction(s), is stated in such a way that the site(s) fall into an area with minimum horizontal deformations, and the coal losses locked in pillars be minimized. The mathematical formalization of the above situation depends on the method used for predicting horizontal deformations.

Prediction procedures can be divided into two main groups on the basis of their fundamental principles, that is, empirical and theoretical. Empirical methods offer certain advantages over theoretical approaches (Salamon, 1993) since computational techniques involved in the former ones are simple, and various interrelated factors can be graphically presented as well (Hoha, 1985).

A new approach for selecting an optimal design solution in case of excavation of gently inclined coal seams is proposed in this paper. Two criteria characterizing the efficiency are simultaneously taken into account:

- 1 Minimum values of the expected horizontal surface deformations during mining activities

- 2 Maximum extraction of the coal deposit

The second criterion provides for minimum losses of coal resources in the protective pillars left under surface site(s). The models derived can easily be extended, and they are independent of the approaches used for predicting horizontal deformations. The choice of an efficient design solution is performed by applying of multi-objective decision making, which can be considered as a set of procedures for finding a certain number of compromise variants from multiple alternatives, while avoiding the evaluation of all possible solutions (Yu, 1989).

The optimization problem then considered is solved by use of a multiple criteria decision method based on the Pareto-optimality (Pareto, 1906). An algorithm is developed, its basic stages are described briefly, and the implementation of the computational procedure is demonstrated with a numerical example.

2: MATHEMATICAL FORMULATION

The method of "typical curves" for predicting the surface movement and deformations has become a popular approach used in Bulgaria. This method utilizes the following input data (see Fig. 1):

- H_m : mining works depth, [m]
- m : mining thickness, [m];
- α : coal seam inclination, [deg];
- $\beta_0, \beta_1, \beta_2$: limit angles, [deg];
- $\alpha_1, \alpha_2, \alpha_3$: angles of full undermining, [deg];
- ϕ : propagation angle of mining influence, [deg];
- k, q : subsidence factors, $k = f(\alpha_3), q = f(\alpha_1, \alpha_2)$.

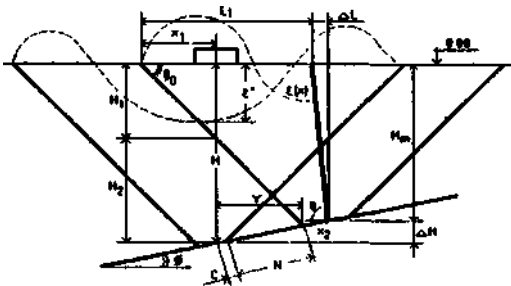


Fig. 1 Parameters of the method of "typical curves": along dip (a), and along strike (b)

Horizontal deformations, $\varepsilon(x)$, [mm/m], are calculated according to the formula:

$$\varepsilon(x) = \frac{W_m}{L_i} F(z) = \frac{m \cdot q \cdot \cos(\phi) \cdot \sqrt{m \cdot n_2}}{L_i} F(z) \quad (1)$$

$i = 1, 2, 3.$

where: W_m : maximum subsidence, [mm]; n_1, n_2 : coefficients depending on the mining depth (H_m) and the lengths of the mining face along dip (D_1) and strike (D_2), respectively. Both coefficients depend on k -subsidence factor ($n_1 \in [0, 1], n_2 \in [1, 1]$); $F(z)$: numerical values of the typical curve empirically obtained through accurate monitoring surveys; L_j ($j = 1, 2, 3$): lengths of semi-subsidence trough along dip (L_1, L_2), and along strike (L_3).

The ordinates of the 8 (x)-curves are calculated for values of $z = x / L_j$ at intervals $0.1 L_j$, where $0 \leq x \leq L_j$, and x is the abscissa interval starting from the maximum subsidence point ($0 \leq x \leq L_j$).

2.1. Applicable conditions for the development of the models

The search for an optimal design solution for the undermining problem is based on the following assumptions

- the number of extracted beds is equal to 1;
- the coal seam inclination is constant;
- the number of working faces is equal to 2;
- the starting moment of mining works is identical for both faces;
- the velocities of movement of both faces are also identical;
- the dimensions and the position of the first working face are fixed.

2.2. Selection of criteria and independent variables

The estimation of characteristics for the project was made on the basis of the following two criteria, i. e. minimum losses of natural resources (J_1), and minimum horizontal deformations (J_2). The selection of the first criterion is dictated by the rapidly increasing importance of coal as a source of energy on a global scale. To some extent, the former criterion represents a technical-economic objective aiming at maximum extraction of the coal deposit, namely, minimum losses of coal in the protective pillars left under surface installations. The second criterion was chosen because of the fact that the tensile and compressive deformations are destructive for the site(s), and the surface damages caused by horizontal deformations are nearly equal in magnitude to the damages caused by the earth surface subsidence (Kratzsch, 1974).

The choice of independent variables was made having in mind the need for simple and yet adequate models. Two main variables were selected (Fig. 2):

• x_1 the distance between the center of the she and the beginning of the basin opening caused by the second mining face;

• x_2 the value of which is equal to half the length of the second mined face along the dip.

Some subsidiary parameters used in expressing both criteria through the independent variables chosen, are also portrayed in Figure 2

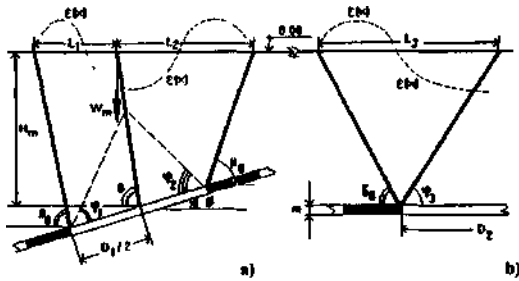


Fig. 2. Graphical plot of the parameters of criteria derived.

2.3. Mathematical formalization of the problem

The following algorithm was used for the mathematical formalization of the above formulated problem:

Step1: Defining the coal pillar (N) as a function of the selected independent variables.

Step2: Determination of horizontal deformations at the centre of the target site as a function of the introduced variables.

Step3: Representation of the J₁ and J₂ criteria by using the defined parameters and variables.

The objective functions derived, and the bounds imposed on the independent variables according to some design considerations (Assenova, 1988), were as follows:

First criterion:

$$J_1 = \left(\frac{H_m - x_1 \cdot \operatorname{tg}(\beta_0)}{\cos(\phi) \cdot [\operatorname{tg}(\beta_0) + \operatorname{tg}(\phi)]} - C \right) \rightarrow \min \quad (2)$$

Second criterion:

$$J_2 = |\varepsilon^* + \varepsilon(x)| \rightarrow \min \quad (3)$$

where:

$$\varepsilon(x) = \frac{\operatorname{tg}(\phi) \cdot F(x) \cdot \sqrt{2k \cdot x / H_m - \operatorname{tg}(\phi) \cdot \left\{ \frac{H_m - x \operatorname{tg}(\beta_0)}{\operatorname{tg}(\beta_0) + \operatorname{tg}(\phi)} + \operatorname{tg}(\phi) \right\}}}{x + \frac{H_m - x \operatorname{tg}(\beta_0)}{\operatorname{tg}(\beta_0) + \operatorname{tg}(\phi)} + \operatorname{tg}(\phi) \cdot \operatorname{ctg}(\phi) \cdot \left\{ \frac{H_m - x \operatorname{tg}(\beta_0)}{\operatorname{tg}(\beta_0) + \operatorname{tg}(\phi)} + \operatorname{tg}(\phi) \right\}}$$

Bounds on the independent variables:

$$0.00 \leq x_1 \leq \operatorname{ctg}(\beta_0) \cdot \left\{ \frac{H_m}{\cos(\phi) \cdot [\operatorname{tg}(\beta_0) + \operatorname{tg}(\phi)]} - C - N \right\}$$

$$20 \leq x_2 \leq 60$$

where C: distance between the centre of the site and the beginning of the protective coal pillar left under the built-up construction (see Fig. 2); N: coal pillar; 8*: horizontal deformation of the site caused by the first mining face; e(x): horizontal deformation of the site caused by the second mining face, $e(x) = f(x_1, X_2)$. The variable HI has integer values only, and it can be specified within other ranges differing from 20 and 60 (Assenova, 1988).

3. OPTIMIZATION STRATEGY

The mathematical problem considered above represents a two-criteria nonlinear optimization task, the solution of which cannot be located by the application of classical search methods involving the optimization of a single function. As distinct from conventional optimization procedures, decision making with multiple criteria permits the determination of a complex extreme that meets the requirements associated with a large number of objectives. Many advanced strategies for multiple criteria decision making have been developed during the past decade (Changkong and Haimes, 1983; Steuer, 1985). In general, the approaches proposed may be divided into two main categories:

1. Methods of vector criterion scalarization.
2. Methods for finding a set of Pareto-optimal solutions.

Using techniques of vector criterion scalarization only one extreme solution may be located even when a set of multiple solutions exists. These approaches are widely favoured by mining engineers and metallurgists, probably because of certain traditions and ease of use, rather than their suitability for solving a given task. Depending on the assumed scheme of compromise in the methods of scalarization, a unique solution that complies with the specifications of some generalized scalar criterion has to be located. Usually, one of the objective functions is adopted as a main criterion, while the rest are applied as constraints (Atipov and Haralampiev, 1986). The basic disadvantage of this approach is related to the inclusion of the limiting conditions. Consequently, the optimum may be found to violate the constraints embedded and the problem cannot be solved. Other variants of the methods of scalarization require a total criterion to be specified. As a rule this criterion represents some mathematical expression (multiplication, summation or a power function) of the objectives

considered. In this way, the problem is reduced to a classical task of (non)linear programming. However, in this kind of optimization problem the extreme frequently coincides with the permissible bounds (upper or lower) of independent variables, which makes finding the exact solution more difficult. In the optimization of a coal processing operation Rubinstein and Volkov (1987) have applied a generalized desirability function that merges all objectives using exponential desirability curves formulated by Harrington (1965). The main disadvantages of the desirability function are:

- excessive calculation time;
- subjective desirability curves;
- occasionally unsuccessful transformation of the various objectives (technological, economic, etc.);
- the irregular sensitivity of the generalized desirability function towards the variations in the objectives.

The methods for finding a set of compromise solutions are based on the concept for optimal control according to the principle proposed by Pareto (1906). A Pareto-optimal condition has the feature that every deviation from it leads to a change for the worse in at least one of m in number objective functions considered to be maximized, i. e., $X^* \in V_x$ is defined as Pareto-optimal if no other solution X^* exists in the permissible area (V_x) such that:

$$F_j(X^*) \geq F_j(X^*), \quad j = 1, 2, \dots, m$$

(the inequality is observed strictly for *one* j at any time).

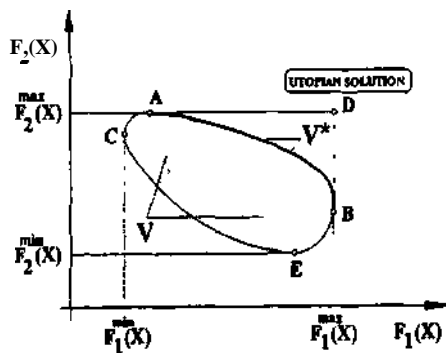


Fig. 3. Pareto-optimal solution of a two-criterial problem.

The set of sub-optimal points (V^*) always lies on the limit of the m -dimensional criteria's space. It

can be seen from the graphical illustration of a two-criteria optimization problem shown in Fig. 3 that a better solution could be found from each point within the permissible region (V) with exception of those points lying on its boundary, i. e., on the curve AB . If different specifications to the objective functions have been posed, e. g., $\min F_1(X)$ and $\max F_2(X)$, the respective Pareto-optimal solutions coincide with the arc AC . Point D (see Fig. 3) is called a Utopian solution and it is only obtainable when the maxima of both criteria are identical.

In spite of some desirable attributes of existing methods for locating a set of Pareto-optimal solutions which have recently been found in many fields of applications, such as in economics, computer-aided design, etc., nothing has been published in mining literature regarding their use. In this paper an approach belonging to the second category of multi-objective optimization strategies is suggested. The optimization procedure proposed here can be considered as a sequence of steps for finding a group of Pareto-optimal points, while avoiding the evaluation of all possible solutions. To this end an algorithm provided by Venkov (1986) was modified by the authors. The procedure was coded in Microsoft FORTRAN version 4.0 for PC and its basic stages are described briefly in the following paragraphs:

Stage I (preliminary search)

The objective functions, constraints imposed on these criteria (if any), and the independent variable bounds are given in an explicit form (no analytic derivatives are required in the algorithm). The search procedure adopted in this stage is conducted as follows:

- (1) M (in number) random trial points are generated consecutively, initially $M = 300$.
- (2) The values of the restrictions are computed. The incompatibility of the imposed constraints is checked. If no constraint is violated, the respective trial is accepted as successful, otherwise the point is eliminated from the search.
- (3) If the permissible area is empty, small concessions in the restrictions are made »id the procedure is continued from step (1) with a number of trials equal to $2M$. When a prescribed number of trials (e. g., $M = 2400$) is reached and no successful one is found, the search is terminated with the message that the stated problem cannot be solved owing to the incompatibility of the constraints.

Stage II (main optimization search)

An optimization strategy for finding a group of Pareto-optimal solutions is applied within the region defined by the set of successful trial points obtained in stage I. The search continues as follows:

- (1) The new permissible area is covered by a grid through the generation of uniformly distributed trial points (see Fig. 4).
- (2) The objective functions are calculated and compared consecutively.
- (3) Since the set of Pareto-optimal solutions lies on the criteria space limit (V^*), the points found at the spatial right angles of a selected trial point are rejected, so that the search becomes progressively crowded towards the criteria bounds converging to the optimum domain - see Figure 4.
- (4) The procedure is iteratively repeated from step (1) until some pre-assigned number of trials has been exceeded or a certain number of consecutive trials fail to identify a new sub-optimal point.

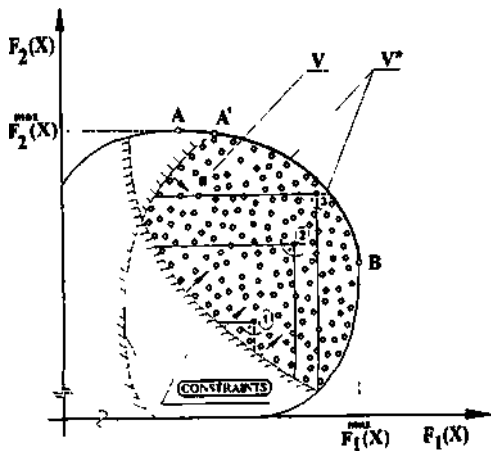


Fig. 4. Graphical presentation of the optimization algorithm.

In the case of two-criteria problems (such as in the example below) it is possible for an approximation of the compromise (Pareto-optimal) curve to be traced and studied thoroughly.

4. EXAMPLE

The computational approach proposed here was tested with a numerical example. The input data were similar to those for an underground coal mine in Bulgaria:

$H_m = 250.00$ [m]; $m = 1.8$ [m]; $\alpha = 15^\circ$ [deg];
 $\beta_0 = 56^\circ$ [deg]; $\alpha_p = 82.5^\circ$ [deg]; $k = 0.70$;
 $q = 0.64$; $C = 50.00$ [m]; $e^* = -1.20$ [mm/m].

The problem of determining the coal working faces' parameters and their optimal position regarding the site being constructed was specified in such a way that the following constraints imposed onto the derived criteria had to be satisfied: $0.5 \leq J_i \leq 80.00$, and $0.0 \leq h \leq 2.00$. The problem of engineering design was then solved with the computer program developed, and a confined set of sub-optimal points was identified.

Some of the results subsequently obtained are presented in Table 1, while a linear-log scale approximation of the compromise curve is portrayed in Fig. 5.

Table 1: Output data of the optimization procedure.

X1	X2	J1	h
89.06	59	19.76	0.3774
96.09	57	13.61	0.7875
108.98	58	2.29	1.3517
84.67	58	23.61	0.1686
101.07	56	9.23	1.0474
22.55	30	78.07	0.0055
83.49	58	24.64	0.1042
99.31	54	10.77	1.0217
95.80	59	13.85	0.7187
110.59	53	0.882	1.5407
81.30	56	26.57	0.0432
93.01	55	16.29	0.6974
98.29	59	11.67	0.8353
81.73	55	26.18	0.0980
93.45	55	15.91	0.7187
91.55	59	17.57	0.5111
103.85	58	6.790	1.1197
86.27	56	22.21	0.3175
79.24	54	28.37	0.0101
102.68	58	7.818	1.0660
98.87	57	11.16	0.9176
73.09	43	3176	0.0073
59.61	22	45.57	0.0069
107.08	59	3.965	1.2383

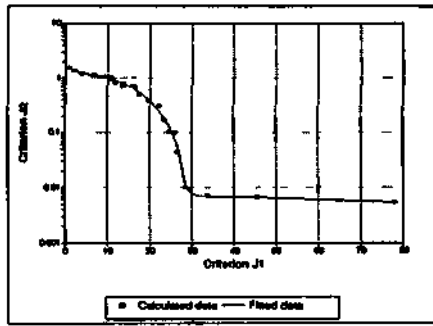


Fig. 5. Approximation of the optimal compromise curve (linear-log scale).

It can be seen from Table 1 that there is no alternative design solution that ensures better values of both criteria than the remaining ones. The results indicate that all the solutions are Pareto-optimal, and that the required specifications are being met at the same time. The output data of the program could be used for a detailed sequential analysis, and the design engineer (decision maker) can choose any of the obtained solution giving a priority or preference to arbitrary considerations. Finally, to prevent an eventual collapse of the coal-pillar, and for re-assessment of its design, the pillar geometries have to be calculated by use of the squat-pillar formula (Wagner and Madden, 1984).

CONCLUSIONS

O. A new approach for solving an important problem of minimizing horizontal surface deformations caused by underground mining activities is proposed.

•. Two criteria characterizing the optimal undermining performance are derived. The first criterion provides for minimum losses of coal resources in the protective pillars left under surface sites, while the second objective ensures minimum values of the expected horizontal deformations during mining activities.

•. The formulated problem is solved by use of a multi-objective optimization method based on the Pareto-optimality. The implementation of this approach is demonstrated with a numerical example. The computational procedure was repeatedly tested, and it proved to be applicable for convex, as well as non-convex compromise curves,

such as in the demonstrated problem of engineering design.

O. The derived models are adequate, and they can be applied along with any other methods predicting horizontal deformations as well.

•. The models can be improved in the future by introducing new parameters (as time factor) and by extending the range of their applicability (for instance: solving of the same problem for two coal seams or three faces, etc.).

A basic advantage of the proposed approach is the cyclic recurrence of the following stages in the engineering design:

- (i) formulation of the problem;
- (ii) mathematical representation (model building) of the problem;
- (iii) optimization - finding a set of effective solutions;
- (iv) analysis of solutions, selection of a variant from the multiple alternatives giving a priority to arbitrary considerations;
- (v) optimal design activities.

The implementation of the described approach is simplified to a great extent by the possibility of using a computer to solve the problem. The computational procedure may be incorporated in a knowledge based system that could facilitate the development of more powerful software for optimal undermining of surface sites.

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